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ABSTRACT

The role of negative instances in the acquisition of the mathematical concepts of distributivity and homomorphism was examined. Two treatment levels for distributivity (positive instances and positive and negative instances) and the same treatment levels for homomorphism were crossed to form a 2 x 2 factorial design with 23 subjects per cell. Criterion variables were number of correct classifications, stimulus intervals, and postfeedback intervals. All pretests, threatments, and posttests were administered using computer terminals. There was a significant effect for treatment levels of distributivity on the correct classifications of new instances of distributivity favoring the treatments containing negative instances (p .05). There was a significant interaction between treatment levels of distributivity and treatment levels of homomorphism on correct classification of new instances of homomorphism (p .05). The results supported the hypotheses that negative instances enhance mathematical concept acquisition and that effects of negative instances for one concept transfer to another concept. (Author/DT)

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HUMAN CONCEPT FORMATION: NEGATIVE INSTANCES

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U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE

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FINAL

REPORT

By

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On HUMAN CONCEPT FORMATION: NEGATIVE INSTANCES

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Submitted by Richard J. Shumway

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Education

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Richard J. Shumway, The Ohio State University

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The role of negative instances in the acquisition of the mathematical concepts of distributivity and homomorphism was examined. Two treatment levels for distributivity (positive instances and positive and negative instances) and the same treatment levels for homomorphism were crossed to form a 2 x 2 factorial design with 23 subjects per cell. Criterion variables were number of correct classifications, stimulus intervals, and postfeedback intervals. All pretests, treatments, and posttests were administered using computer terminals. There was a significant effect for treatment levels of distributivity on the correct classifications of new instances of distributivity favoring the treatments containing negative instances ($p < .05$). There was a significant interaction between treatment levels of distributivity and treatment levels of homomorphism on correct classification of new instances of homomorphism ($p < .05$). The results supported the hypotheses that negative instances enhance mathematical concept acquisition and that effects of negative instances for one concept transfer to another concept.

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INTRODUCTION

Mathematical concepts, while complex in nature, are precisely defined and can therefore be useful for research in concept acquisition. The focus was the role of negative instances in mathematical concept acquisition. Two questions were investigated:

1. What are the different effects of an instructional sequence of all positive instances and a sequence of positive and negative instances on the acquisition of the concepts of distributivity and/or homomorphism?

2. Do effects for negative instances on the acquisition of one concept transfer to the acquisition of another concept?

Background

Negative instances have been considered by mathematicians to be essential to the understanding of advanced mathematical concepts (Gelbaum and Olmsted, 1964; Steen and Seebach, 1970). Dienes (1964) argues for the use of negative instances in the teaching of mathematics to elementary and secondary school children.

Educational psychologists have stated explicitly that all instructional sequences designed for concept learning should include negative instances (Bereiter and Engelman, 1966; Markle and Tiemann, 1970, 1972).

A review of the research in experimental psychology generally supported a deleterious effect for negative instances in conjunctive concept learning, but that for nonconjunctive concepts the use of negative instances was sometimes advantageous (Clark, 1971; Bourne and Dominowski, 1972).

Instructional Research

Research in mathematical concept acquisition generally supports the use of negative instances to improve concept acquisition. In a classroom study, Shumway (1971) found that negative instances discouraged overgeneralization errors by 13 and 14 year old Ss for concepts involving the properties of binary operations. Using programmed instruction, Marine (1972) found results favoring negative instances and Dossey (1972) found deleterious effects for negative instances. Shumway (1972), using computer terminals to present treatments for commutativity and associativity to 14 and 15 year old Ss, not only found results favoring treatments containing negative instances, but also found that the effect for negative instances transferred from one concept to another. Results in poetry and linguistics have also supported the inclusion of negative instances in instructional sequences (Tennyson, Woolley and Merrill, 1971; Markle and Tiemann, 1972).

Laboratory Research

Several studies involving concepts defined over finite universal classes using the dimensions of color, size, and shape have been carried out to investigate the role of positive and negative instances in the formation of simple concepts. In reporting one of the earliest of such studies, Smoke (1933) states that although there were no significant differences found in the rate at which concepts were learned when series of positive instances were compared with mixed series of both positive and negative instances, there was some evidence that negative instances tended to discourage 'snap judgments' on the part of the subjects during the learning of difficult concepts.

Hovland (1952) noted that the relative size of the class of positive instances and the class of negative instances introduced variability into the amount of information a particular instance communicated. Even when the amount of information instances communicated was equated, Hovland and Weiss (1953) found that more subjects completed their task successfully when the instances were positive than when the instances were a mixture of positive and negative instances or when the instances were all negative. Bruner, Goodnow, and Austin (1956) and others support the results of Hovland and Weiss (Glanzer, Huttenlocher, and Clark, 1963; Haygood and Devine, 1967; Mayzner, 1962).

Research has also shown that subjects seem to have both an inability and an unwillingness to use negative instances (Bruner, et al., 1956, Dominowski, 1968; Donaldson, 1959; Wojtaszek, 1971). It has been suggested that negative verbal information is simply more difficult for subjects to handle (Donaldson, 1959; Gough, 1965; Johnson-Laird and Tagart, 1969; Tavrow, 1966; Wales and Grieve, 1969; Wason, 1959). Others have suggested that the strategies used by the subjects do not seem to be compatible with effective use of negative instances (Bruner, et al., 1956; Braley, 1963; Conant and Trabasso, 1964; Davidson, 1969; Denny and Benjafield, 1969; Dervin and Deffenbacher, 1970; Duncan, 1964; Eifermann and Steinitz, 1971; Gough, 1965; Huttenlocher, 1967; O'Neill, 1969; Tagatz, et al., 1968; Wickelgren and Cohen, 1962).

In general support of the theory that subjects are unfamiliar with the use of negative instances, Freibergs and Tulving (1961) have shown that although initial differences in median time to solution between subjects using positive instances and subjects using negative instances favor subjects using positive instances, these differences are virtually nonexistent at the end of a 20-problem series. The results of Chlebek and Dominowski (1970), Fryatt and Tulving (1963), Haygood and Stevenson (1967), Tavrow (1966), and Weber and Woodward (1966) appear to support the contention that subjects can learn to use negative instances as effectively as positive instances.

It has also been pointed out by Bruner et al. (1956) that the nature of the rule defining the concept is related to the role which negative instances play. Conant (1966) reports that for disjunctive concepts, mixed series of positive and negative instances are favored over an all positive series. Huttenlocher (1962, 1964) reports that for one dimensional concepts, a mixed series of positive and negative series is favored over any other. Bourne and Guy (1968) report that in addition to confirming the results of Hovland and Weiss (1953) concerning the role of negative instances in conjunctive concept formation and the results of Hunt and Hovland (1960) concerning the relative difficulty of conjunctive, disjunctive, and conditional concept formation, it was also found that for rule learning (attributes given, rule unknown), subjects performed best on all rules when a mixture of positive and negative instances was presented.

Recent research suggests that there is an interaction between usefulness of negative instances and the nature of the conceptual rule (Bourne and Ekstrand, 1969; Fraunfelker, 1971; Giambra, 1969; Krebs, 1970; Schvaneveldt, 1966; Shore and Sechrest, 1961; Taplin, 1971; Weber and Woodward, 1966).

In summary, it appears that for simple unidimensional and conjunctive concepts a sequence of positive instances is favored. For more complicated rule learning negative instances are favored. It has been hypothesized by Bourne, Ekstrand, and Dominowski (1971) that "the most important consideration for understanding how people use stimulus information is, in all probability, the number of different instances contained in the positive and negative categories (p. 265)."

Discussion

In all of the investigations cited from the laboratory, the concepts studied were simple concepts defined over a finite universal class. It would seem appropriate to investigate concepts where the universal class was infinite and the concepts were of the more complicated type encountered in the study of mathematics. Wason (1960) reports that in using semi-mathematical concepts defined over an infinite universal class, the successful students were the ones who could suggest negative instances to test in the process of discovering a concept.

A second observation concerning the investigations cited is that the training sequence is somewhat different than that encountered in school learning. The subjects were given a series of instances and were asked to discover the concept by examining the instances. It is common procedure in school learning to first present the subject with the definition of the concept and then examine a series of instances in order to learn the concept (Ausubel, 1968). Frase (1972) supports this point by making a distinction between concept definition and concept formation.

Markle and Tiemann (1972) and Gagné (1965) suggest that in school learning the subject is to classify new instances which have not previously been seen. Clark (1971) describes the critical differences between the concept attainment tasks of experimental research and concepts in the classroom and calls for research in the content areas to bridge the gap. Cronbach (1957), Pereboom (1971), and Tagatz, Meinke, and Lemke (1968) have also called for studies in specific content areas.

It is the purpose of this investigator to borrow freely from the work of experimental psychologists and attempt to develop controlled studies concerning the role of negative instances in the acquisition of mathematical concepts.

A previous study by this investigator (Shumway, 1972) has confirmed that for the concepts of commutativity and associativity, negative instances improved Ss performance on associativity and that the effect for negative instances transferred from one concept to another. Several alternate explanations for the results were proposed. There were near significant results for stimulus intervals and postfeedback intervals ($p < .07$). Differences in time variables could account for the advantage cited for negative instances. The Ss were not remarkably successful on the criterion measure. Differences could be attributed to the Ss maintaining the same proportion of positive and negative instances during criterion measure as was present during the treatment and simply guessing. The results may be unique to the concepts of commutativity and associativity. It was proposed that a similar study be conducted with different concepts and Ss of an age older than 14 and 15 to further investigate these questions. The current study meets the above criteria.

Definitions and Hypotheses

The first requisite is a definition for concept. As Kendler (1964) has pointed out, none of the many definitions of concept given in the literature can be considered totally adequate since they do not characterize such concepts as color, time, space, etc. For the purposes of an experimental study, the most useful definitions are those given by Hovland (1952), Kendler (1961), Hunt (1962), Bourne (1966), and others in the field of experimental psychology. We choose to use components of the definitions of Kendler and Hunt.

A concept is a partitioning of a class X into two disjoint classes X_1 and X_2. The elements of the class X_1 are called positive instances of the concept and elements of the class X_2 are called negative instances of the concept. The class X is called the universal class over which the concept is defined. The notion of partition requires that the class X be the union of the class X_1 and the class X_2 and that the classes X_1 and X_2 are disjoint. To say that a subject knows the concept over the class X is to say that given any object from the class X the subject is able to identify the object as a member of the class X_1 or the class X_2

associated with the concept over the class X. For example, a student knows the concept pencil over the class of all objects in a particular schoolroom: if given any object from the schoolroom, the student is able to identify the object as either a pencil (an element of X_1) or a non-pencil (an element of X_2). The definition requires that this can be done with every object in the given schoolroom (every element of X). In analyzing the definition formally, it is clear that one need only be able to identify the elements of two of the three classes X, X_1, and X_2 in order to know the concept over the class X. This follows from the results that

$$X = X_1 \cup X_2, \quad X_1 = X - X_2, \quad \text{and} \quad X_2 = X - X_1$$

This observation would seem to suggest three possible strategies for the maximally efficient learning of a given concept over a class X with partition X_1, X_2. Namely, the student would learn to identify the classes

$$X_1 \text{ and } X_2, \text{ or } X \text{ and } X_2, \text{ or } X \text{ and } X_1.$$

If one allows for redundancy, a fourth strategy can be added; the student would learn to identify the classes

$$X, X_1, \text{ and } X_2.$$

Negative instances play a role in three of the four strategies.

The major problem investigated was the effect that negative instances have on the formation of mathematical concepts and if that effect transfers from one concept to another. Two mathematical concepts over the infinite class of all binary operations on the set of rational numbers and elementary functions were used. Concept A was distributivity of a binary operation over another binary operation and Concept B was homomorphism of a function over a binary operation. All treatments for the acquisition of Concepts A and B were a series of instances which the subject was required to classify as a positive or negative instance of the concept. The subject was told the relevant attribute. The treatment for Concept A consisting of only positive instances was denoted by A+, the treatment consisting of positive and negative instances was denoted by A±. The treatments B+ and B± were defined similarly. Figure 1 specifies the four treatment groups.

	A+	A±
B+	A+B+	A±B+
B±	A+B±	A±B±

Figure 1. Treatment groups

The class X for both Concepts A and B is infinite. Each of the four treatments specified in the above table consisted of 20 instances, 10 for Concept A and 10 for Concept B. In the cases where both positive and negative instances were presented, 5 were positive and 5 were negative. The treatments were administered by IBM 2741 computer terminals.

The following definitions were made:

Binary operation: A binary operation * on a set S is a correspondence which associates with every ordered pair (a,b) of elements of S a unique element a * b of S.

Distributive: A binary operation @ on a set S is said to be distributive over a binary operation o on S if and only if for every a,b, and c in S,

$$a \mathbin{@} (b \circ c) = (a \mathbin{@} b) \circ (a \mathbin{@} c), \text{ and}$$
$$(a \circ b) \mathbin{@} c = (a \mathbin{@} c) \circ (b \mathbin{@} c).$$

Function: A function f from a set X to a set Y is a correspondence which associates with each element x of X a unique element f(x) of Y.

Homomorphism: Let X and Y be sets with binary operations · and * respectively. A homomorphism from X to Y is a function f from X to Y such that for all x_1 and x_2 in X,

$$f(x_1 \cdot x_2) = f(x_1) * f(x_2).$$

Postfeedback interval: The postfeedback interval is the length of time between the presentation of the feedback and the occurrence of the next stimulus.

Stimulus interval: The stimulus interval is the length of time the stimulus is available to the subject for inspection.

A sample instance during the treatment was as follows:

Stimulus:

1. a @ b = 3 * a * b   2 @ 5 = 30,   4 @ 1 = 12.

   a o b = a + b   4 o 7 = 11,   6 o 2 = 8.

   a @ (b o c) = (a @ b) o (a @ c) ?

Response:

y

Feedback:

Correct.

Response:

Hit 'return key' to receive next stimulus.

The stimulus interval was taken to be the length of time between the end of the typing of the stimulus, i.e., the symbol "?," and the entering of the symbol "y," the response. There was no delay of the informative feedback. As soon as the response was entered, the feedback was typed. The postfeedback interval was taken to be the length of time between the typing of the feedback and the subject's hitting of the return key to receive the next stimulus.

The following hypotheses were tested:

There is no significant interaction or main effects for levels of A (A+,A±) and levels of B (B+,B±) in

I. Mean number of correct identifications for

   a) criterion measure for Concept A;
   b) criterion measure for Concept B.

II. Mean total stimulus interval and/or postfeedback interval during

a) treatment for Concept A;
b) treatment for Concept B;
c) criterion measure for Concept A;
d) criterion measure for Concept B.

Formal Analysis of the Conceptual Tasks

In order to increase the usefulness of the research, the concepts of distributivity and homomorphism are formally analyzed below. Both concepts were shown to be infinite conjunctive concepts defined over infinite classes. The paradigm was the reception paradigm, the task was classified as rule or principle learning rather than attribute learning, and the major criterion variable was the number of correct classifications of new instances.

Let X be the infinite class of all binary operations defined on the set of rational numbers, Ra. Let P(a,b,c) be the statement:

$$a @ (b \circ c) = (a @ b) \circ (a @ c),$$

where the operation o is either addition, +, or multiplication, *.[a] A binary operation, @ is said to be a positive instance of Concept A if and only if:

$$\bigwedge_{a,b,c \in Ra} P(a,b,c),$$

where

$$\bigwedge_{a,b,c \in Ra} P(a,b,c) = P(a_1,b_1,c_1) \wedge P(a_2,b_2,c_2) \wedge \ldots P(a_i,b_i,c_i) \wedge \ldots$$

and a_i, b_i, and c_i range over Ra, the set of rational numbers.

[a] From a mathematician's point of view, the definition is restrictive. We have, in fact, restricted the concept to left-distributivity rather than left- and right-distributivity and we are considering distributivity over only two (+,*) of a possible infinite number of binary operations.

Thus, in theory at least, in order to determine whether or not an operation satisfied Concept A a subject would need to show that $P(a,b,c)$ was true for every possible value of a, b, and c. That is, show that $P(1,2,1)$, $P(1,2,3)$, $P(1/3,3,6)$, ... , etc. were all true; hence, Concept A was classified as an infinite conjunctive concept. However, it is likely that subjects would infer that $\bigwedge_{a,b,c \in Ra} P(a,b,c)$ were true from a single case such as $P(1,2,3)$. Such a strategy would not fail for the binary operations chosen for the treatments (See Appendix A.4).

The paradigm was the reception paradigm, the instances were given to the subjects in a predetermined order.

After each instance was presented, the subject was asked whether or not $P(a,b,c)$ was true for the given instance. The task was judged not to be attribute learning since the subject was told the relevant attribute to test after each instance was presented. It is possible that one should consider such tasks as concept definition as Frase has suggested (Frase, 1972). However, without further analysis the concept was classified as a form of rule or principle learning.

The major criterion variable was the number of correct classifications of new instances. Thus, it was not possible for subjects to simply memorize which binary operations satisfied Concept A. The operations used for the criterion measure formed instances which had not been classified by any of the subjects before.

The analysis for Concept B was similar. Let X be the infinite class of all function defined on the set of rational numbers, Ra. Let $Q(a,b,c)$ be the statement:

$$f(a \circ b) = f(a) \circ f(b)$$

where the operation o is either addition, +, or multiplication, *. A function, f is said to be a positive instance of Concept B if and only if:

$$\bigwedge_{a,b,c \in Ra} Q(a,b,c).$$

Thus, Concept B was also classified as an infinite conjunctive concept. As with Concept A, the paradigm was the reception paradigm, the task was classified as rule or principle learning and the major criterion variable was the number of correct classifications of new instances.

METHODS

Subjects, Design, and Treatments

A random sample of 92 elementary education majors enrolled in a required mathematics course at Ohio State University were randomly assigned in equal numbers to four treatments. The course was the second of two mathematics courses designed to explore the mathematical concepts taught in elementary school. Most subjects were college sophomores or juniors.

Concept A was defined to be distributivity of a binary operation over a binary operation and Concept B was defined to be homomorphism of a function over a binary operation. The symbol A+ denoted a treatment of 10 positive instances of Concept A and the symbol A± denoted a treatment of 5 positive instances and 5 negative instances of Concept A. The symbols B+ and B± were defined similarly. Figure 2 specifies the 2 x 2 design matrix. Each treatment consisted of 20 instances in a fixed but random order. Each of the four treatments is given in full in Appendix A.4.

	A+	A±
B+	A+B+	A±B+
B±	A+B±	A±B±

Figure 2. Design matrix

Table I specifies the number and type of instances for each treatment.

TABLE I

NUMBER AND TYPE OF TREATMENT INSTANCES PER CELL
AND NUMBER OF SUBJECTS PER CELL

Cell	Treatment	Number and Type of Instances				Total	Number of Subjects Per Cell
		Concept A		Concept B			
		Positive	Negative	Positive	Negative		
11	A+B+	10	0	10	0	20	23
12	A±B+	5	5	10	0	20	23
21	A+B±	10[a]	0	5	5	20	23
22	A±B±	5[a]	5	5	5	20	23

[a]Item twelve for cell 21 and item 3 for cell 22 were scored as positive instances and the subjects received feedback which identified the instance as positive. In fact, however, the instance was actually negative. An examination of the response patterns revealed no discernible disruptive influence. It was assumed that the programming error would reduce the chances for different effects for treatments. The item was treated as a positive instance in the analysis.

A sample instance from treatment A+B+ was as follows:

Stimulus:

1. f(x) = 6 * x, f(4) = 24, f(7) = 42.

   a o b = a + b 3 o 9 = 12, 6 o 2 = 8.

   f(a o b) = f(a) o f(b) ?

Response:

n

Feedback:

Incorrect.

Response:

'Return key'.

Instruments

The treatments were administered with an IBM 360/50 computer and IBM 2741 computer terminals. The programming language was Coursewriter III, version 2 (IBM, 1969). Stimulus intervals and postfeedback intervals for each item were recorded as well as the student's responses. Sample student interaction and computer programming for each treatment are displayed in Appendix A.4.

Each of the binary operations and functions used for the posttests (POA and POB) were used for the calculational pretests PCA and PCB. Stimulus intervals were recorded during both pretests. The items for PCA and PCB were randomly ordered. Sample items of PCA and PCB were as follows:

Stimulus:

4. $a @ b = 3 * a \qquad 5 @ 2 = 15, \qquad 7 @ 9 = 21.$

   $a \circ b = a * b, \qquad 5 \circ 4 = 20, \qquad 6 \circ 7 = 42.$

   $4 @ (3 \circ 2) = ?$ (PCA)

Response:

12

Stimulus:

5. $f(x) = 0, \qquad f(9) = 0, \qquad f(2) = 0.$

   $a \circ b = a + b, \qquad 3 \circ 8 = 11, \qquad 4 \circ 2 = 6.$

   $f(4 \circ 7) = ?$ (PCB)

Response:

2

A complete list of items for the pretests PCA and PCB are given in Appendix A.3. The pretests were designed to determine whether or not subjects could successfully calculate with binary operations and functions. The calculations were of the type necessary to classify instances during the treatments and the posttests. Table II gives reliability estimates for the 92 subjects of the experiment. Neither PCA nor PCB proved to have high reliability estimates. Mean scores in excess of 9 out of a possible 10 were to suggest that the usual reliability estimates were inappropriate.

TABLE II

RELIABILITY ESTIMATES FOR PRETESTS

Instrument	Number of Items	Reliability Estimate[a]
PCA	10	.56
PCB	10	.35

PCA - Pretest, Calculations with operations.
PCB - Pretest, Calculations with functions.
[a]Kuder-Richardson Formula 20 reliability estimate.

The posttests for Concept A and Concept B, POA and POB, each consisted of 10 instances not in any of the treatments. Sample items of POA and POB were as follows:

Stimulus:

1. f(x) = x, f(6) = 6, f(3) = 3.

   a o b = a + b, 2 o 7 = 9, 6 o 1 = 7.

   f(a o b) = f(a) o f(b) ?

Response:

y

Stimulus:

2. a @ b = 2 * a * b 3 @ 7 = 42, 4 @ 2 = 16.

   a o b = a * b 5 o 3 = 15, 7 o 9 = 63.

   a @ (b o c) = (a @ b) o (a @ c) ?

Response:

y

Stimulus intervals were recorded for all items of POA and POB. A complete list of items for the posttest POA and POB may be found in

Appendix A.5. Table III gives the number and type of instances for POA and POB as well as the reliability estimates of .65 and .58.

TABLE III

NUMBER OF INSTANCES, TYPE OF INSTANCES, AND RELIABILITY ESTIMATES OF POSTTESTS FOR CONCEPT A AND CONCEPT B (POA AND POB)

Instrument	Number of Instances			Reliability Estimate[a]
	Positive	Negative	Total	
POA	5	5	10	.65
POB	5	5	10	.58

[a]Kuder-Richardson Formula 20 reliability estimate.

Figure 3 gives a flow chart of the complete experimental sequence. There were three sessions at the computer terminal. The first, lasting approximately 15 minutes, consisted of an introduction to binary operations and functions and the pretests PCA and PCB. The second session, lasting approximately 20 minutes, consisted of a brief introduction and one of the four experimental treatments. The third session, lasting approximately 15 minutes, consisted of the two posttests, POA and POB. Approximately two-thirds of the Ss completed all three sessions in one setting. In all cases Ss completed all three sessions within seven days. Computer terminals were available at many locations on campus and Ss could use any available terminal between 8 a.m. and 11 p.m.

For the experiment, the independent variables were:

1. Levels of A (A+ or A±);
2. Levels of B (B+ or B±);

3. PCA - Pretest, calculations with operations;
4. PCB - Pretest, calculations with functions;

5. PCSIA - Total Stimulus Interval for PCA;
6. PCSIB - Total Stimulus Interval for PCB;

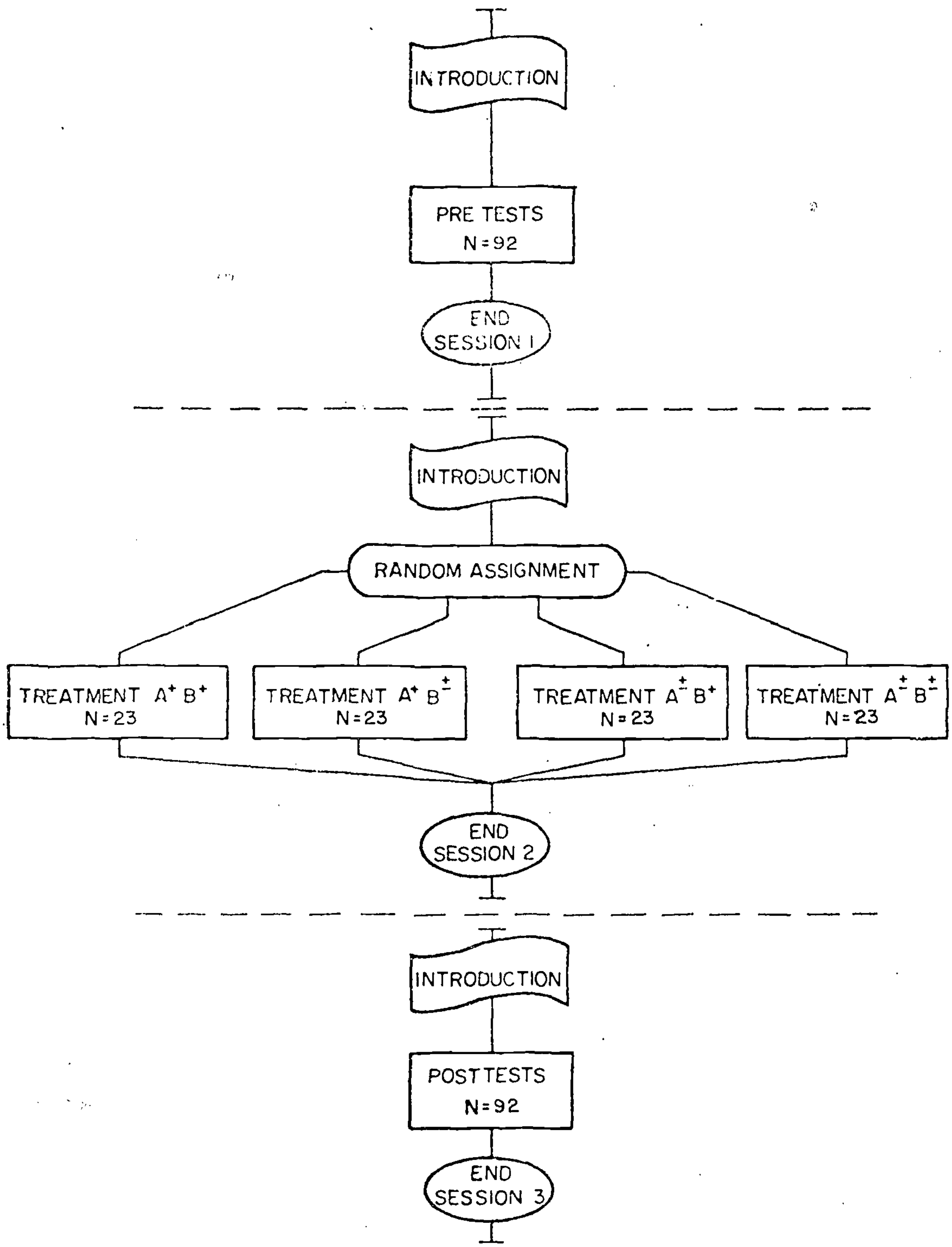


Figure 3. Flow chart of experiment

and the dependent variables were

1. POA - Posttest for Concept A;
2. POB - Posttest for Concept B;

3. POSIA - Total Stimulus Interval for POA;
4. POSIB - Total Stimulus Interval for POB;

5. TSIA - Total Stimulus Interval during Treatment for Concept A;
6. TSIB - Total Stimulus Interval during Treatment for Concept B;

7. TPIA - Total Postfeedback Interval during Treatment for Concept A;
8. TPIB - Total Postfeedback Interval during Treatment for Concept B;

The design may be diagrammed as follows:

$$
\begin{array}{llll}
R_1 & O_1^A O_2^A O_1^B O_2^B & (X_1 \; O_3^A O_4^A O_3^B O_4^B) & O_5^A O_6^A O_5^B O_6^B \\
R_2 & O_1^A O_2^A O_1^B O_2^B & (X_2 \; O_3^A O_4^A O_3^B O_4^B) & O_5^A O_6^A O_5^B O_6^B \\
R_3 & O_1^A O_2^A O_1^B O_2^B & (X_3 \; O_3^A O_4^A O_3^B O_4^B) & O_5^A O_6^A O_5^B O_6^B \\
R_4 & O_1^A O_2^A O_1^B O_2^B & (X_4 \; O_3^A O_4^A O_3^B O_4^B) & O_5^A O_6^A O_5^B O_6^B
\end{array}
$$

where R_1-R_4 are the four groups, X_1-X_4 are the four treatments and O_i^A and O_i^B are defined as follows:

$$
\begin{array}{lll}
O_1^A \text{ - PCA} & O_3^A \text{ - TSIA} & O_5^A \text{ - POA} \\
O_2^A \text{ - PCSIA} & O_4^A \text{ - TPIA} & O_6^A \text{ - POSIA} \\
O_1^B \text{ - PCB} & O_3^B \text{ - TSIB} & O_5^B \text{ - POB} \\
O_2^B \text{ - PCSIB} & O_4^B \text{ - TPIB} & O_6^B \text{ - POSIB}
\end{array}
$$

Thus, given the 2 x 2 design matrix

	A+	A±
B+	A+B+	A±B+
B±	A+B±	A±B±

The following hypotheses were tested: There is no significant interaction or main effects for levels of A and levels of B in:

I. Achievement: mean performance on

a) POA;
b) POB;

II. Time: mean total on

a) TSIA;
b) TSIB;
c) TPIA;
d) TPIB;
e) POSIA;
f) POSIB.

Analysis

The data were analyzed using the Clyde MANOVA program (Clyde, 1969) for a multivariate two-way analysis of covariance. Because of the symmetry of the design the results for Concept B were viewed as a potential replication for the results for Concept A. Hence, the analysis for Concept B was done separately from the analysis for Concept A. Achievement variables were separated from time variables.

RESULTS

Pretest Analysis

Table IV summarizes the means and standard deviations for each cell on the pretest measures related to Concept A. The means of over 9 out of a maximum of 10 indicated that with new binary operations, Ss were quite skillful at calculations of the kind necessary to classify instances during the treatment and the posttests.

TABLE IV

CELL MEANS AND STANDARD DEVIATIONS FOR PRETESTS RELATED TO CONCEPT A (PCA AND PSIA)

Cell	Treatment	Statistic	PCA	PSIA[a]
11	A+B+	M	9.435	225.174
		SD	0.945	88.415
12	A±B+	M	9.087	270.652
		SD	1.240	161.227
21	A+B±	M	9.304	226.348
		SD	1.020	104.273
22	A±B±	M	8.913	221.609
		SD	1.535	118.234

M - Mean, SD - Standard Deviation
[a]Stimulus intervals are in seconds.

The pretests PCA and PSIA were subjected to multivariate and univariate analysis of variance to determine if significant differences existed between groups on measures related to Concept A before the treatments. Table V summarizes the results of the analysis. No p-values were less than .14.

TABLE V

MULTIVARIATE AND UNIVARIATE ANALYSIS OF VARIANCE OF PRETESTS RELATED TO CONCEPT A (PCA AND PSIA)

Variable(s)	Test	Source	df	F	p <
PCA, PSIA	M	A x B	2,87	0.531	.590
PCA	U		1,88	0.007	.931
PSIA	U		1,88	0.989	.323
PCA, PSIA	M	A	2,87	1.202	.306
PCA	U		1,88	2.157	.146
PSIA	U		1,88	0.651	.422
PCA, PSIA	M	B	2,87	0.769	.467
PCA	U		1,88	0.366	.547
PSIA	U		1,88	0.898	.356

M - Multivariate test, U - Univariate test

Table VI summarizes the means and standard deviations for each cell on the pretest measures related to Concept B. The means of over 9 out of a maximum of 10 suggest that, with functions, Ss were quite successful at calculations of the kind necessary to classify instances during the treatment and the posttests.

TABLE VI

CELL MEANS AND STANDARD DEVIATIONS FOR PRETESTS RELATED TO CONCEPT B (PCB AND PSIB)

Cell	Treatment	Statistic	PCB	PSIB[a]
11	A+B+	M	9.348	219.044
		SD	0.935	81.729
12	A±B+	M	9.174	241.000
		SD	1.072	122.795
21	A+B±	M	9.522	209.522
		SD	0.730	80.512
22	A±B±	M	9.130	187.609
		SD	1.058	86.637

M - Mean, SD - Standard Deviation
[a]Stimulus intervals are in seconds

The pretests PCB and PSIB were subjected to a multivariate and univariate analysis of variance to determine if significant differences existed between groups on measures related to Concept B before the treatments. Table VII summarizes the results of the analysis. No p-values were less than .11.

TABLE VII

MULTIVARIATE AND UNIVARIATE ANALYSIS OF VARIANCE OF PRETESTS RELATED TO CONCEPT B (PCB AND PSIB)

Variable(s)	Test	Source	df	F	p <
PCB, PSIB	M	A x B	2,87	0.848	.432
PCB	U		1,88	0.296	.588
PSIB	U		1,88	1.238	.269
PCB, PSIB	M	A	2,87	1.004	.371
PCB	U		1,88	1.999	.161
PSIB	U		1,88	0.000	.999
PCB, PSIB	M	B	2,87	1.266	.287
PCB	U		1,88	0.106	.745
PSIB	U		1,88	0.547	.114

M - Multivariate test, U - Univariate test

While no pretest differences were significant, covariance procedures were chosen for the analysis to increase the power of the tests and because there was a clear conceptual relationship between pretests and posttests.

Achievement, Concept A (POA)

Table VIII gives the unadjusted cell means and standard deviations for the posttest related to Concept A (POA). The maximum possible score was 10. The expected score by chance alone was 5. The range of cell means from 7.5 to 8.6 suggested that Ss had some skill in classifying instances for Concept A.

TABLE VIII

CELL MEANS AND STANDARD DEVIATIONS FOR POSTTEST RELATED TO CONCEPT A (POA)

Cell	Treatment	Statistic	POA
11	A+B+	M	7.565
		SD	2.085
12	A±B+	M	8.565
		SD	1.502
21	A+B±	M	8.261
		SD	1.982
22	A±B±	M	8.522
		SD	1.592

M - Mean, SD - Standard Deviation

The variable POA, as the major criterion variable for Concept A, was analyzed using a univariate analysis of covariance with PCA as covariate. Table IX summarizes the results of the analysis of POA. There was a significant effect for levels of A (A+, A±) favoring A± ($p < .05$).

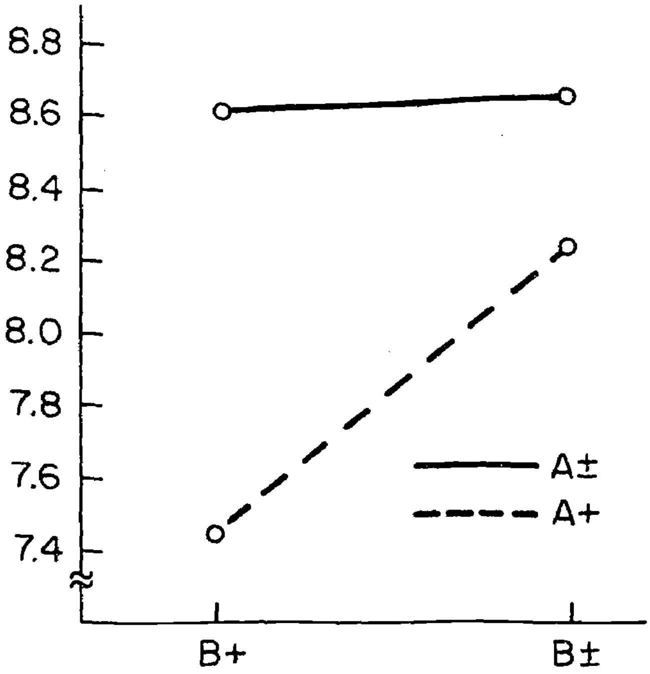
TABLE IX

ANALYSIS OF COVARIANCE FOR CONCEPT A OF POA USING PCA AS COVARIATE

Source	df	F	p <
Equality of Regression	3,84	1.246	.298
Regression	1,87	5.407	.022*
A x B	1,87	0.967	.328
A	1,87	4.216	.043*
B	1,87	1.070	.304

POA - Posttest for Concept A.
PCA - Pretest, Calculations without parentheses.
*$p < .05$

Figure 4 displays the adjusted cell and margin means and a plot of the cell means.



	A+	A±	
B+	7.475	8.601	8.638
B±	8.218	8.620	8.419
	7.846	8.610	

Figure 4. Adjusted means for POA (A effect)

Achievement, Concept B (POB)

Table X gives the cell means and standard deviations for the post-test for Concept B (POB). The maximum possible score was 10. The expected score by chance alone was 5. The range of cell means from 7.8 to 8.8 suggested that Ss had some skill in classifying instances for Concept B.

TABLE X

CELL MEANS AND STANDARD DEVIATIONS FOR POSTTEST RELATED TO CONCEPT B (POB)

Cell	Treatment	Statistic	POB
11	A+B+	M	7.826
		SD	1.922
12	A±B+	M	8.783
		SD	1.278
21	A+B±	M	8.739
		SD	1.453
22	A±B±	M	8.174
		SD	1.614

M - Mean, SD - Standard Deviation

The variable POB, as the major criterion variable for Concept B, was analyzed using a univariate analysis of covariance with PCB as covariate. Table XI summarizes the results of the analysis of POB. There was a significant interaction between levels of A and levels of B ($p < .05$).

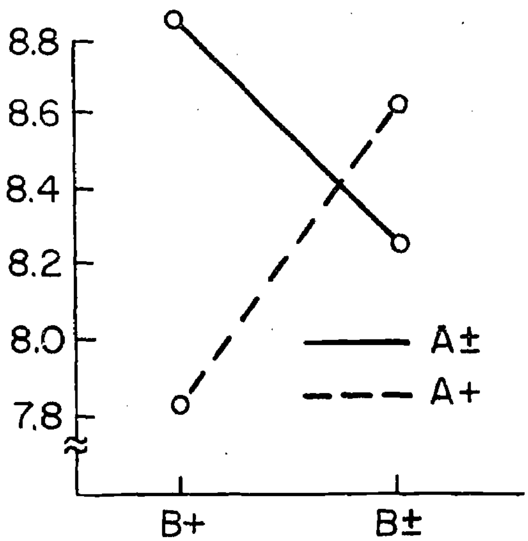
TABLE XI

ANALYSIS OF COVARIANCE FOR CONCEPT B OF POB USING PCB AS COVARIATE

Source	df	F	p <
Equality of Regression	3,84	2.364	.077
Regression	1,87	10.501	.002**
A x B	1,87	4.984	.028*
A	1,87	1.208	.275
B	1,87	.138	.711

POB - Posttest for Concept B
PCA - Pretest, Calculations without parentheses
PCB - Pretest, Calculations with parentheses
$^{**}p < .01$, $^{*}p < .05$

Figure 5 displays the adjusted cell and margin means and a plot of the cell means. The interaction was tested for disordinality. While a t-test showed that the cell mean for A+B+ was significantly lower than the cell mean for A±B ($t = 2.03$, $p < .05$), there was no evidence that the cell mean for A+B± was significantly higher than the cell mean for A±B± ($t = 1.24$, $p > .2$). Hence, the interaction effect was not a disordinal interaction. It appears that negative instances for Concept A improved performance on Concept B when no negative instances for Concept B were present. Transfer occurred.



	A+	A±	
B+	7.797	8.847	8.322
B±	3.615	8.262	8.439
	8.206	8.555	

Figure 5. Adjusted means for FOB (A x B effect)

<u>Time, Concept A (PCSIA, TSIA, TPIA, POSIA)</u>

Table XII summarizes the means and standard deviations for each cell on the time variables for Concept A.

TABLE XII

CELL MEANS AND STANDARD DEVIATIONS FOR TIME VARIABLES[a] RELATED TO CONCEPT A (PCSIA, TSIA, TPIA, AND POSIA)

Cell	Treatment	Statistic	PCSIA	TSIA	TPIA	POSIA
11	A+B+	M	225.174	180.696	36.304	311.826
		SD	88.415	119.710	43.100	200.262
12	A±B+	M	270.652	262.609	31.565	374.652
		SD	161.227	137.029	40.689	187.501
21	A+B±	M	226.348	254.131	42.174	312.304
		SD	104.273	133.420	57.559	170.533
22	A±B±	M	221.609	239.391	31.696	296.826
		SD	118.234	155.185	37.075	196.393

M - Mean, SD - Standard Deviation
PCSIA - Pretest, Calculations with operations Stimulus Interval
TSIA - Treatment Stimulus Interval for Concept A.
TPIA - Treatment Postfeedback Interval for Concept A.
POSIA - Posttest Stimulus Interval for Concept A.
[a]All time variables are in seconds.

The time variables were subjected to a multivariate analysis of covariance. Appendix A.7 reports the correlation matrix for all variables of the study. The significant correlations ($p < .05$) among some of the time variables support the use of multivariate analysis. (See Table XVII, Appendix A.7). The total stimulus interval on the pretest (PCSIA) was used as the covariate. Table XIII summarizes the results of the analysis of the time variables for Concept A. There was no evidence of any cell differences on any of the time variables.

TABLE XIII

MULTIVARIATE AND UNIVARIATE ANALYSIS OF COVARIANCE FOR TIME VARIABLES RELATED TO CONCEPT A (TSIA, TPIA, AND POSIA) USING PCSIA AS COVARIATE

Variable(s)	Test	Source	df	F	p <
TSIA, TPIA, POSIA	M	Equality of	9,199.7	0.647	.756
TSIA	U	Regression	3,84	0.812	.226
TPIA	U		3,84	1.049	.375
POSIA	U		3,84	0.965	.413
TSIA, TPIA, POSIA	M	Regression	3,85	3.653	.016*
TSIA	U		1,87	10.835	.001**
TPIA	U		1,87	0.456	.501
POSIA	U		1,87	2.715	.103
TSIA, TPIA, POSIA	M	A x B	3,85	0.674	.571
TSIA	U		1,87	2.039	.157
TPIA	U		1,87	0.053	.819
POSIA	U		1,87	0.679	.412
TSIA, TPIA, POSIA	M	A	3,85	0.729	.537
TSIA	U		1,87	0.910	.343
TPIA	U		1,87	0.737	.393
POSIA	U		1,87	0.214	.644
TSIA, TPIA, POSIA	M	B	3,85	1.963	.126
TSIA	U		1,87	1.571	.213
TPIA	U		1,87	0.147	.703
POSIA	U		1,87	0.672	.414

*p < .05, **p < .01

Time, Concept B (PCSIB, TSIB, TPIB, AND POSIB)

Table XIV summarizes the means and standard deviations for each cell on the time variables for Concept B.

TABLE XIV

CELL MEANS AND STANDARD DEVIATIONS FOR TIME VARIABLES[a] RELATED TO CONCEPT B (PCSIB, TSIB, TPIB, AND POSIB)

Cell	Treatment	Statistic	PCSIB	TSIB	TPIB	POSIB
11	A+B+	M	219.044	279.261	32.609	298.391
		SD	81.729	167.451	27.729	156.370
12	A±B+	M	241.000	430.696	40.696	308.174
		SD	122.795	315.983	42.160	133.735
21	A+B±	M	209.522	370.043	48.087	270.826
		SD	80.512	212.860	57.656	140.895
22	A±B±	M	187.609	328.391	75.522	239.348
		SD	86.637	201.000	103.817	155.051

M - Mean, SD - Standard Deviation
PCSIB - Pretest, Calculations with functions Stimulus Interval.
TSIB - Treatment Stimulus Interval for Concept B.
TPIB - Treatment Postfeedback Interval for Concept B.
POSIB - Posttest Stimulus Interval for Concept B.
[a]All time variables are in seconds.

As in the Concept A analysis, the time variables were subjected to a multivariate analysis of covariance. The total stimulus interval on the pretest (PCSIB) was used as the covariate. Table XV summarizes the results of the analysis of the time variables for Concept B. None of the multivariate p-values were less than .11. Since the multivariate test for homogeneity of regression was not significant, the univariate tests for homogeneity were ignored and deemed not significant. Consistent with the analysis for Concept A, there was no evidence of any cell differences on any of the time variables ($p < .05$).

TABLE XV

MULTIVARIATE AND UNIVARIATE ANALYSIS OF COVARIANCE FOR TIME VARIABLES RELATED TO CONCEPT B (TSIB, TPIB, AND POSIB) USING PCSIB AS COVARIATE

Variable(s)	Test	Source	df	F	p <
TSIB, TPIB, POSIB	M	Equality of Regression	9,199.7	1.184	.307
TSIB	U		3,84	0.731	.536
TPIB	U		3,84	0.418	.740
POSIB	U		3,84	3.521	.018*
TSIB, TPIB, POSIB	M	Regression	3,85	3.059	.033*
TSIB	U		1,87	6.884	.010**
TPIB	U		1,87	0.265	.608
POSIB	U		1,87	6.781	.011*
TSIB, TPIB, POSIB	M	A x B	3,85	1.630	.189
TSIB	U		1,87	3.049	.084
TPIB	U		1,87	0.596	.442
POSIB	U		1,87	0.147	.702
TSIB, TPIB, POSIB	M	A	3,85	2.023	.117
TSIB	U		1,87	1.384	.243
TPIB	U		1,87	1.728	.192
POSIB	U		1,87	0.134	.715
TSIB, TPIB, POSIB	M	B	3,85	2.023	.117
TSIB	U		1,87	0.101	.751
TPIB	U		1,87	3.693	.058
POSIB	U		1,87	1.358	.247

*p < .05, **p < .01

Summary

Table XVI summarizes the results of the analyses for Concept A and Concept B. For Concept A, there was a significant A effect on achievement (POA) favoring treatments of both positive and negative instances.

TABLE XVI

SUMMARY OF ANALYSES FOR CONCEPT A AND CONCEPT B FOR WHICH STATISTICALLY SIGNIFICANT DIFFERENCES WERE FOUND ($p < .05$)

Variable(s)	Effects	p <	Level Favored +	Level Favored ±	Transfer Effect
Concept A					
POA	A	.043		Yes	
Concept B					
POB	A x B	.028	-	Yes	Yes

For Concept B there was a significant interaction (A x B) effect. It appeared that the treatment including negative instances of Concept A improved performance on Concept B, but there was no evidence that negative instances for both concepts improved performance on Concept B.

CONCLUSIONS

Two questions were examined.

1. What are the different effects of an instructional sequence of all positive instances and a sequence of positive and negative instances on the acquisition of the concepts of distributivity and/or homomorphism?

2. Do effects for negative instances on the acquisition of one concept transfer to the acquisition of another concept?

Question 1 was answered as follows:

For the acquisition of the concept of distributivity a sequence of positive and negative instances was favored over a sequence of all positive instances.

Question 2 was answered as follows:

There was an interaction effect between negative instances for distributivity and negative instances for homomorphism on the acquisition of homomorphism. The effect of negative instances for distributivity improved performance on homomorphism when no negative instances for homomorphism were present. Transfer occurred.

Discussion

In a study of similar design using the concepts of commutativity and associativity, Shumway reported that negative instances improved performance and that the effects of negative instances transferred from one concept to another (Shumway, 1972). However, it was not clear that there was not a difference in the time variables of stimulus interval and postfeedback interval which could have accounted for the differences found. Subjects performed at a level no better than that expected by chance alone and did not show marked ability with the pretest calculations.

Several of the limitations of the study by Shumway (1972) were not found in this study. There were no significant multivariate or univariate time differences. It is clear that subjects were not guessing. Subjects performance was at a level better than expected by chance alone and Ss exhibited a great deal of ability with the pretest calculations. Nevertheless, the results again supported the conclusion that a treatment of negative and positive instances improved concept acquisition and that the effect of negative instances transferred from one concept to another.

The results support the research strategy taken by this author. Negative instances have been shown to be an important variable in laboratory concept acquisition (Clark, 1972). For unidimensional and conjunctive concepts negative instances are deleterious. For disjunctive, conjunctive, and biconditional concepts negative instance enhance concept acquisition. In order to generalize such results to concepts recognized for their social value; for example, concepts in the school curriculum, it is necessary to perform studies which attempt to replicate the laboratory results with concepts from the school curriculum. It appears that although distributivity and homomorphism can be classified as conjunctive or possibly even unidimensional concepts, the increase in the size of the class over which the concept is defined to infinity and the modification of the instructional sequence to actually giving the subject the attribute to test sufficiently complicates the task so that, contrary to laboratory evidence, negative instances improve subjects performance. Replicative studies are urged.

Three separate modifications are suggested by the results.

M1. Manipulate the size of the class, X, over which the concept is defined.

M2. Reduce the size of the set, Ra, over which each binary operation is defined.

M3. Remove the instruction which told the subject the relevant attribute of the operation.

Each of these possible modifications changes the conceptual task to one less similar to classroom instruction. Since most mathematical concepts are defined over infinite classes, M1 would make the task less like the comparable classroom task. Most, but not all number systems are infinite, so M2 would also make the task less like the comparable classroom task. Since most classroom concepts are learned with some directions as to the relevant attributes, M3 would also make the task less like the comparable classroom task. However, such modifications may, in fact, increase the generalizability of the results because they allow the experimenter to measure the effect of each variable while keeping constant the effect of the others. The author would favor such systematic modifications of the experiment in order to attempt to identify the reasons for the effects for negative instances which have been observed.

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A.2 GLOSSARY OF TERMS

1. Associative: A binary operation * on a set S is said to be associative if and only if for every a, b, and c in S,
   $a * (b * c) = (a * b) * c.$

2. Attributes: Discernable characteristics of an object, event, or idea that distinguish it from other objects, events, or ideas.

3. Bioconditional: A statement is a bioconditional if and only if it is of the form "____ if and only if ____."

4. Binary operation: A binary operation * on a set S is a correspondence which associates with every ordered pair (a,b) of elements of S a unique element a * b of S.

5. Commutative: A binary operation * on a set S is said to be commutative if and only if for every a and b in S,
   $a * b = b * a.$

6. Concept: A concept over the class X is a partitioning of a class X into two disjoint classes X_1 and X_2.

7. Concept acquisition: Concept acquisition tasks are those concept learning tasks where a simple set of instructions would not produce the same behavior as the conventional training procedures. (Kendler, H. H., 1964, p. 227)

8. Concept identification: Concept identification tasks are those concept learning tasks where instructions could produce the same behaviors as the conventional training procedures. (Kendler, H. H., 1964, p. 227)

9. Conditional: A statement is said to be a conditional if and only if it is of the form "If ____, then ____."

10. Conjunction: A statement is said to be a conjunction if and only if it is of the form "____ and ____."

11. Coursewriter III: Coursewriter III is an interactive computer language designed for computer-assisted instruction (CAI).

12. Delay of informative feedback: The delay of informative feedback is the length of time between the subject's response to a question and the presentation of the feedback associated with the subject's response.

13. Disjunction: A statement is said to be a disjunction if and only if it is of the form "____ and/or ____."

14. Distributive: A binary operation @ on a set S is said to be distributive over a binary operation o on S if and only if for every a, b, and c in S,

$$a \,@\, (b \circ c) = (a \,@\, b) \circ (a \,@\, c), \text{ and}$$
$$(a \circ b) \,@\, c = (a \,@\, c) \circ (b \,@\, c).$$

15. Function: A function f from a set X to a set Y is a correspondence which associates with each element x of X a unique element f(x) of Y.

16. Homomorphism: Let X and Y be sets with binary operations $\cdot$ and $*$ respectively. A homomorphism from X to Y is a function f from X to Y such that for all x_1 and x_2 in X,

$$f(x_1 \cdot x_2) = f(x_1) * f(x_2).$$

17. Negative instance: Given a concept over a class X with partition X_1, X_2 and given that the elements of X_1 are examples of the concept, then the negative instances of the concept are the elements of X_1.

18. Positive instances: Given a concept over a class X with partition X_1, X_2 and given that the elements of X_1 are examples of the concept, then the positive instances of the concept are the elements of X_1.

19. Postfeedback interval: The postfeedback interval is the length of time between the presentation of the feedback and the occurrence of the next stimulus.

20. Rational numbers: The set of rational numbers Ra is the set of all quotients of the form a/b where a and b are intergers and $b \neq 0$.

21. Rules: Conceptual rules are statements which specify how the relevant attributes are combined for use in classifying an instance. Simple rules are typically characterized as conjunctive, disjunctive, conditional, or bioconditional depending on the form of the defining statement.

22. Stimulus interval: The stimulus interval is the length of time the stimulus is available to the subject for inspection.

23. Terminal: The word terminal will be used to refer to the IBM 2741 computer terminal.

24. Universal class: The universal class is the class over which a concept is defined.

A.3 INTRODUCTION AND PRETEST

INTRODUCTION

Welcome to computer-assisted-instruction. This message is being transmitted to you from the Ohio State University computer center.

Whenever you are asked to answer a question, simply type your answer and then hit the RETURN key.

OK! Let's see if you understand these directions. What is your first name?

frank

Very good, Frank.

You have successfully communicated with a computer. Remember that after you finish typing an answer you must depress the RETURN key.

The purpose of the time hou will spend communicating with the computer is to teach you some mathematics. It is important for you to be careful and do as well as you can. This will help to determine the usefulness of computers for teaching mathematics.

Before we can begin, we neef to discuss the meaning of the symbols and notation which will be used.

You should be familiar with the following symbols:

SYMBOL	MEANING
+	ADD
-	SUBTRACT
*	MULTIPLY
/	DIVIDE

Notice that the symbol, *, means multiply. We will not use the symbol, x, to mean multiply.

Please hit the RETURN key when you are ready to continue.

In the unit you will be studying we will be using some new symbols in addition to the symbols: +, -, *, and /.

Two of these symbols are o and @. These symbols will be used to define several different ways of combining numbers. Here are some examples. Think of <u>a</u> and <u>b</u> as representing any two integers.

a o b = a + b + 1.

Then 2 o 4 = 7, and 5 o 3 = 9.

So, 3 o 4 = ?

13

No. 3 o 4 = 3 + 4 + 1. What is 3 + 4 + 1 ?

8

Excellent. Here's one using the symbol, @.

a @ b = a*b - 2.

Then 2 @ 3 = 4, and 6 @ 4 = 22.

So, 5 @ 2 = ?

5

b. a @ b = a*b - 2 so
5 @ 2 = 5*2 - 2. Now you can finish it.

8

Good!!

Let's do a problem that is a little different.

Supposd we define @ and o as follows:

a @ b = 2*a + b,	3 @ 1 = 7,	4 @ 5 = 13.
a o b = a*b,	4 o 3 = 12,	2 o 6 = 12.

3 @ (2 o 5) = ?

10

No. You must be careful on problems which involve parentheses. You must remember to find the value of the part which is inside the parentheses first. Then you do the other part. Try again.

16

Excellent!! You catch on very quickly.

Remember that any problem involving parentheses must be done with great care. Do the part inside the parentheses first, then find the final answer.

There is one other bit of notation with which you must familiarize yourself. This notation involves the symbol, f(x). An example may best illustrate how this notation is used.

Suppose we define f(x) = 2*x - 1. Then,

f(3) = 2*3 - 1

6 - 1 = 5. So, f(3) = 5.

What is f(5) ?

6

Notice that we have defined f(x) in a certain way.

f(x) = 2*x - 1

If we substitute 5 for x we have

f(5) = 2*5 - 1. Now what is f(5)?

9

OK! You have the right idea.

Define $f(x)$ as before. That is, $f(x) = 2*x - 1$.

Also, let $a \circ b = a + b$. What is $f(4 \circ 2)$?

7

No. You must calculate the value of the expression inside the parentheses first. Try again.

11

Excellent!!!

Let's define $f(x)$ as follows:

$f(x) = x + 8$, $f(2) = 10$, $f(6) = 14$.

What is $f(4)$?

12

Very good!

Suppose $f(x) = x + 8$ and $a \circ b = a*b$.

Then $f(1) \circ f(3) = ?$

99

Good.

In the next section, keep in mind that @ and o may be defined in many different ways. Also, $f(x)$ may have a different meaning from problem to problem.

We are coming to a very important part of your session with the computer for today. You will be asked to answer several problems in order to see how well you understand what has been discussed so far.

The computer will tell you how o and @ are defined or how o and f(x) are defined. Two calculational examples will be given and then you will be asked to answer 2 specific examples. Regardless of your answer the computer will continue on8to the next problem. Please answer each question carefully so that we will be able to determine how well you understand this material.

When you are ready to start hit the RETURN key.

PRETESTS (PCA AND PCB)

Sample Student Interaction

1. f(x) = 7*x, f(6) = 42, f(4) = 28.

a o b = a + b, 3 o 5 = 8, 8 o 5 = 13.

f(1 o 2) = ?

21

a o b = a + b, 2 o 7 = 9, 6 o 1 = 7.

f(3 o 1) = ?

4

a o b = a*b, 3 o 4 = 12, 5 o 1 = 5.

f(1) o f(2) = ?

8

$01

4. a @ b = 3*a, 5 @ 2 = 15, 7 @ 9 = 21.

a o b = a*b, 5 o 4 = 20, 6 o 7 = 42.

4 @ (3 o 2) = ?

12

5. f(x) = 0, f(9) = 0, f(2) = 0.

a o b = a + b, 3 o 8 = 11, 4 o 2 = 6.

f(4 o 7) = ?

Sample Programming

```
  pretes
    1-  0 pr
    1-  1 ld 0/c1
$   1-  2 ld 0/c2
    2-  0 cm /pre/1.  f(x) = 7*x,       f(6) = 42,      f(4) = 28,
    2-  1 cc /     a o b = a + b,      3 o 5 = 8,      8 o 5 = 13.
    2-  2 cc /     f(1 o 2) = ?/21
&   3-  0 qu 1.  f(x) = 7*x,       f(6) = 42,      f(4) = 28.

&   3-  1        a o b = a + b,      3 o 5 = 8,      8 o 5 = 13.

&   3-  2        f(1 o 2) = ?

&   3-  3 ca 21
    3-  4 cb 21
&   3-  5 ld c0/c3
&   3-  6 ad 1/c1
&   3-  7 ty

&   3-  8 un

&   3-  9 ld c0/c3
&   3- 10 ad 1/c2
&   3- 11 br pr
    4-  0 cm /pre/2.  f(x) = x,        f(6) = 6,       f(3) = 3.
```

```
   10-  0 qu 4.    a @ b  =  3*a,           5 @ 2  =  15,        7 @ 9  =  21.

&  10-  1     -    a o b  =  a*b,           5 o 4  =  20,        6 o 7  =  42.


&  10-  2          4 @ (3 o 2)  =  ?


&  10-  3 ca 12
   10-  4 cb 12
&  10-  5 ld c0/c3
&  10-  6 ad 1/c1
&  10-  7 ty


&  10-  8 un


&  10-  9 ld c0/c3
&   10- 10 ad 1/c2
&  10- 11 br pr
```

Pretest items

			Concept A	Concept B
1. f(x) = 7*x,	f(6) = 42,	f(4) = 28.		
a o b = a + b,	3 o 5 = 8,	8 o 5 = 13.		
f(1 o 2) = ?				21
2. f(x) = x	f(6) = 6,	f(3) = 3.		
a o b = a + b,	2 o 7 = 9,	6 o 1 = 7.		
f(3 o 1) = ?				4
3. f(x) = x^3,	f(3) = 27,	f(2) = 8.		
a o b = a*b,	3 o 4 = 12,	5 o 1 = 5.		
f(1) o f(2) = ?				8
4. a @ b = 3*a,	5 @ 2 = 15,	7 @ 9 = 21.		
a o b = a*b,	5 o 4 = 20,	6 o 7 = 42.		
4 @ (3 o 2) = ?			12	
5. f(x) = 0,	f(9) = 0,	f(2) = 0.		
a o b = a + b,	3 o 8 = 11,	4 o 2 = 6.		
f(4 o 7) = ?				0
6. a @ b = sqr(a),	4 @ 8 = 2,	9 @ 5 = 3.		
a o b = a + b,	1 o 7 = 8,	8 o 4 = 12.		
(16 @ 3) o (9 @ 4) = ?			7	
7. f(x) = 4*x,	f(8) = 32,	f(2) = 8.		
a o b = a*b,	3 o 8 = 24,	6 o 2 = 12.		
f(2) o f(1) = ?				32
8. f(x) = sqr(3*x),	f(3) = 3,	f(12) = 6.		
a o b = a*b,	4 o 7 = 28,	5 o 2 = 10.		
f(1 o 3) = ?				3
9. a @ b = 0,	4 @ 8 = 0,	7 @ 5 = 0.		
a o b = a*b,	5 o 3 = 15,	2 o 8 = 16.		
(2 @ 6) o (5 @ 1) = ?			0	

				Concept A	Concept B
10.	f(x) = 1,	f(7) = 1,	f(4) = 1.		
	a o b = a*b,	3 o 6 = 18,	5 o 2 = 10.		
	f(4) o f(6) = ?				1
11.	f(x) = 3*x^2,	f(2) = 12,	f(3) = 27.		
	a o b = a + b,	4 o 5 = 9,	6 o 2 = 8.		
	f(1 o 3) = ?				48
12.	a @ b = 2*a*b,	3 @ 7 = 42,	4 @ 2 = 16.		
	a o b = a + b,	5 o 6 = 11,	6 o 1 = 7.		
	(3 @ 1) o (2 @ 4) = ?			22	
13.	f(x) = x - 1,	f(9) = 8,	f(4) = 3.		
	a o b = a + b,	2 o 7 = 9,	6 o 1 = 7.		
	f(5) o f(4) = ?				7
14.	a @ b = 3,	7 @ 2 = 3,	4 @ 9 = 3.		
	a o b = a*b,	3 o 4 = 14,	8 o 9 = 72.		
	7 @ (4 o 6) = ?			3	
15.	a @ b = 1,	5 @ 2 = 1,	6 @ 9 = 1.		
	a o b = a*b,	3 o 7 = 21,	5 o 3 = 15.		
	(5 @ 7) o (2 @ 4) = ?			1	
16.	a @ b = 5*a*b,	3 @ 2 = 30,	4 @ 1 = 20.		
	a o b = a + b,	3 o 6 = 9,	8 o 4 = 12.		
	2 @ (3 0 2) = ?			50	
17.	f(x) = 3,	f(5) = 3,	f(1) = 3.		
	a o b = a*b,	2 o 7 = 14,	6 o 3 = 18.		
	f(7) o f(4) = ?				9
18.	a @ b = b,	4 @ 9 = 9,	8 @ 2 = 2.		
	a o b = a + b,	5 o 9 = 14,	3 o 8 = 11.		
	6 @ (3 o 5) = ?			8	

				Concept A B
Pretest-(cont.)				
19.	$a @ b = 3*b$,	2 @ 8 = 24,	6 @ 4 = 12.	
	$a \circ b = a + b$,	6 o 3 = 9,	5 o 7 = 12.	
	$8 @ (2 \circ 4) = ?$			18
20.	$a @ b = a^2$,	3 @ 5 = 9,	2 @ 7 = 4.	
	$a \circ b = a + b$,	6 o 2 = 8,	8 o 9 = 17.	
	$(3 @ 2) \circ (4 @ 1) = ?$			25

A.4 TREATMENTS

INTRODUCTION

We are beginning session 2, Frank.
During this session we will be looking at several different ways of defining o, @, and f(x). Once again you will be given a definition of o and @ or a definition of o and f(x). Two calculational examples will follow. Your task is to answer one question for each set of definitions. The question will require a simple 'yes' or 'no' answer. In fact you won't even have to type the whole word. If your answer is 'yes' type 'y' and if your answer is 'no' type 'n.' After you type your answer you will be told if it is CORRECT or INCORRECT. The computer will then wait until you depress the RETURN key before continuing on to the next problem. It is important that you remember to hit the RETURN key when you are ready to go on to the next problem. The computer will wait for you to signal it to go on so that you can study the problem you have just answered. In this way you will be given a chance to decide why your answer was CORRECT or INCORRECT.
Let's do two sample problems.

SAMPLE 1.	a @ b = a*b,	2 @ 3 = 6,	6 @ 4 = 24.
	a o b = a + b,	4 o 1 = 5,	5 o 7 = 12.
	a @ (b o c) = (a @ b) o (a @ c) ?		

y
Correct.

SAMPLE 2. f(x) = x*x, f(2) = 4, f(3) = 9.

a o b = a*b, 4 o 5 = 20, 8 o 3 = 24.

f(a o b) = f(a) o f(b) ?

n

Incorrect.

You had some trouble with one of the sample problems. Try to be very careful when you work the problems which follow.

TREATMENT A+B+

<u>Sample Student Interaction</u>

1. a @ b = 3*a*b, 2 @ 5 = 30, 4 @ 1 = 12.

a o b = a + b, 4 o 7 = 11, 6 o 2 = 8.

a @ (b o c) = (a @ b) o (a @ c) ?

y

Correct.

5. f(x) = 6*x, f(4) = 24, f(7) = 42.

a o b = a + b, 3 o 9 = 12, 6 o 2 = 8.

f(a o b) = f(a) o f(b) ?

y

Correct.

TREATMENT A+B+ (continued)

Sample Programming

```
    1-  0 cm /dis/1.   a @ b  =  3*a*b,        2 @ 5  =  30,        4 @ 1  =  12.
    1-  1 cc /   * a o b  =  a + b,          4 o 7  =  11,        6 o 2  =  8./y/n
&   2-  0 qu 1.   a @ b  =  3*a*b,          2 @ 5  =  30,        4 @ 1  =  12.
&   2-  1


&   2-  2        a o b  =  a + b,          4 o 7  =  11,        6 o 2  =  8.
&   2-  3



&   2-  4        a @ (b o c)  =  (a @ b) o (a @ c) ?


&   2-  5 ca y
&   2-  6 ld c0/c3
&   2-  7 ad 1/c1
&   2-  8 ty        Correct.



&   2-  9 wa n
&   2- 10 ld c0/c4
&   2- 11 ad 1/c2
&   2- 12 ty        Incorrect.



&   2- 13 br pr
&   2- 14 un        Please type either 'y' or 'n.'


&   3-  0 rd

&   3-  1 ep
&   3-  2 ld c0/c5
&   3-  3 ad c3/c6
&   3-  4 ad c4/c7
&   3-  5 ld 0/c3
&   3-  6 ld 0/c4


   13-  0 cm /hom/5.   f(x)  =  6*x,          f(4)  =  24,        f(7)  =  42.
   13-  1 cc /      a o b  =  a + b,        3 o 9  =  12,        6 o 2  =  8./y/n
&  14-  0 qu 5.   f(x)  =  6*x,           f(4)  =  24,        f(7)  =  42.
&  14-  1
```

TREATMENT A+B+ (continued)

Sample Programming (continued)

```
& 14-  2         a o b = a + b,     3 o 9 = 12,     6 o 2 = 8.
& 14-  3

& 14-  4         f(a o b) = f(a) o f(b) ?

& 14-  5 ca  y
& 14-  6 ld  c0/c3
& 14-  7 ad  1/c1
& 14-  8 ty        Correct.

& 14-  9 wa  n
& 14- 10 ld  c0/c4
& 14- 11 ad  1/c2
& 14- 12 ty        Incorrect.

& 14- 13 br  pr
& 14- 14 un        Please type either 'y' or 'n.'

& 15-  0 rd

& 15-  1 ep
& 15-  2 ld  c0/c5
& 15-  3 ad  c3/c6
& 15-  4 ad  c4/c7
& 15-  5 ld  0/c3
& 15-  6 ld  0/c4
```

TREATMENT A±B+

Sample Student Interaction

```
1.   f(x)  =  0,          f(2)  =  0,          f(7)  =  0.

     a o b  =  a + b,     3 o 8  =  11,        6 o 1  =  7.


     f(a o b)  =  f(a) o f(b) ?

y
     Correct.


                  2
6.   a @ b  =  b ,        3 @ 4  =  16,        1 @ 5  =  25.

     a o b  =  a*b,       6 o 2  =  12,        4 o 7  =  28.

     a @ (b o c)  =  (a @ b) o (a @ c) ?
y
     Correct.
```

Sample Programming

```
 treat2
   1-  0 cm /hom/1.  f(x)  =  0,         f(2)  =  0,        f(7)  =  0.
   1-  1 cc /      a o b  =  a + b,    3 o 8  =  11,       6 o 1  =  7./y/n
&  2-  0 qu 1.  f(x)  =  0,        f(2)  =  0,        f(7)  =  0.
&  2-  1

&  2-  2        a o b  =  a + b,    3 o 8  =  11,       6 o 1  =  7.
&  2-  3

   2-  4        f(a o b)  =  f(a) o f(b) ?
&

&  2-  5 ca y
&  2-  6 ld c0/c3
&  2-  7 ad 1/c1
&  2-  8 ty       Correct.
```

TREATMENT A±B+ (continued)

Sample Programming (continued)

```
&    2-  9 wa n
&    2- 10 ld c0/c4
&    2- 11 ad 1/c2
&    2- 12 ty        Incorrect.


&    2- 13 br pr
&    2- 14  un       Please type either 'y' or 'n.'


&    3-  0 rd

&    3-  1 ep
&    3-  2 ld c0/c5
&    3-  3 ad c3/c6
&    3-  4 ad c4/c7
&    3-  5 ld 0/c3
&    3-  6 ld 0/c4
     4-  0 cm /hom/2.   f(x)  =  0,        f(8)  =  0,        f(3)  =  0.


     16-  0 cm /dis/               2
                      6.   a  @ b  =  b ,      3 @ 4  =  16,      1 @ 5  =  25.
     16-  1 cc /       a o b  =  a*b,     6 o 2  =  12,      4 o 7  =  28./y/n
 &   17-  0 qu                   2
                  6.   a @ b  =  b ,      3 @ 4  =  16,      1 @ 5  =  25.
 &   17-  1


 &   17-  2            a o b  =  a*b,     6 o 2  =  12,      4 o 7  =  28.
 &   17-  3


 &   17-  4            a @ (b o c)  =  (a @ b) o (a @ c) ?


 &   17-  5 ca y
 &   17-  6 ld c0/c3
 &   17-  7 ad 1/c1
 &   17-  8 ty         Correct.


 &   17-  9 wa n
 &   17- 10 ld c0/c4
 &   17- 11 ad 1/c2
 &   17- 12 ty         Incorrect.
```

TREATMENT A±B+ (continued)

Sample Programming (continued)

```
& 17- 13 br pr
& 17- 14 un       Please type either 'y' or 'n.'

& 18-  0 rd

& 18-  1 ep
& 18-  2 ld c0/c5
& 18-  3 ad c3/c6
& 18-  4 ad c4/c7
& 18-  5 ld 0/c3
& 18-  6 ld 0/c4
TYPE COMMAND
```

TREATMENT A+B±

Sample Student Interaction

```
1.   a @ b  =  1,         3 @ 9  =  1,         5 @ 2  =  1.

     a o b  =  a*b,       4 o 7  =  28,        3 o 2  =  6.


     a @ (b o c)  =  (a @ b) o (a @ c) ?

y

     Correct.


4.   f(x)  =  3*x,        f(4)  =  12,         f(7)  =  21.

     a o b  =  a*b,       3 o 5  =  15,        6 o 2  =  12.


     f(a o b)  =  f(a) o f(b) ?
```

TREATMENT A+B± (continued)

Sample Programming

```
  treat3
    1-  0 cm /dis/1.   a @ b  =  1,        3 @ 9  =  1,         5 @ 2  =  1.
    1-  1 cc /    a o b   =  a*b,        4 o 7  =  28,        3 o 2  =  6./y/n
&   2-  0 qu 1.  a @ b  =  1,        3 @ 9  =  1,         5 @ 2  =  1.
&   2-  1


&   2-  2        a o b  =  a*b,      4 o 7  =  28,        3 o 2  =  6.
&   2-  3



&   2-  4        a @ (b o c)  =  (a @ b) o (a @ c) ?


&   2-  5 ca y
&   2-  6 ld c0/c3
&   2-  7 ad 1/c1
&   2-  8 ty       Correct.



&   2-  9 wa n
&   2- 10 ld  c0/c4
&   2- 11 ad 1/c2
&   2- 12 ty       Incorrect.



&   2- 13 br pr
&   2- 14 un       Please type either 'y' or 'n.'


&   3-  0 rd

&   3-  1 ep
&   3-  2 ld c0/c5
&   3-  3 ad c3/c6
&   3-  4 ad c4/c7
&   3-  5 ld  0/c3
&   3-  6 ld 0/c4
    4-  0 cm /dis/2.   a @ b  =  3*a*b,       2 @ 8  =  48,        6 @ 2  =  36.
```

Sample Programming (continued)

```
   10-  0 cm /hom/4.   f(x)  =  3*x,            f(4)  =  12,          f(7)  =  21.
   10-  1 cc /     a o b  =  a*b,           3 o 5  =  15,         6 o 2  =  12./n/y
&  11-  0 qu 4.   f(x)  =  3*x,           f(4)  =  12,          f(7)  =  21.
&  11-  1

&  11-  2         a o b  =  a*b,          3 o 5  =  15,         6 o 2  =  12.
&  11-  3

&  11-  4         f(a o b)  =  f(a) o f(b) ?

&  11-  5 ca n
&  11-  6 ld c0/c3
&  11-  7 ad  1/c1
&  11-  8 ty      Correct.

&  11-  9 wa y
&  11- 10 ld c0/c4
&  11- 11 ad 1/c2
&  11- 12 ty      Incorrect.

&  11- 13 br pr
&  11- 14 un      Please type either 'y' or 'n.'

&  12-  0 rd

&  12-  1 ep
&  12-  2 ld c0/c5
&  12-  3 ad c3/c6
&  12-  4 ad c4/c7
&  12-  5 ld 0/c3
&  12-  6 ld 0/c4
```

TREATMENT A±B±

Sample Student Interaction

```
1.   f(x)  =  x,          f(4)  =  4,          f(3)  =  3.

     a o b  =  a*b,       2 o 8  =  16,        5 o 3  =  15.


     f(a o b)  =  f(a) o f(b) ?

y

     Correct.


3.   a @ b  =  a,         2 @ 4  =  2,         7 @ 3  =  7.

     a o b  =  a + b,     6 o 3  =  9,         2 o 9  =  11.


     a @ (b o c)  =  (a @ b) o (a @ c) ?

y

     Correct.
```

Sample Programming

```
  treat4
    1-  0 cm /hom/1.   f(x)  =  x,          f(4)  =  4,         f(3)  =  3.
    1-  1 cc /     a o b  =  a*b,         2 o 8  =  16,        5 o 3  =  15./y/n
&   2-  0 qu 1.   f(x)  =  x,          f(4)  =  4,         f(3)  =  3.
&   2-  1

&   2-  2          a o b  =  a*b,       2 o 8  =  16,        5 o 3  =  15.
&   2-  3

&   2-  4          f(a o b)  =  f(a) o f(b) ?
```

TREATMENT A±B± (continued)

Sample Programming (continued)

```
&    2-  5 ca y
&    2-  6 ld c0/c3
&    2-  7 ad 1/c1
&    2-  8 ty      Correct.


&    2-  9 wa n
&    2- 10 ld c0/c4
&    2- 11 ad 1/c2
&    2- 12 ty      Incorrect.


&    2- 13 br  pr
&    2- 14 un      Please type either 'y' or 'n.'


&    3-  0 rd

&    3-  1 ep
&    3-  2 ld c0/c5
&    3-  3 ad c3/c6
&    3-  4 ad c4/c7
&    3-  5 ld 0/c3
&    3-  6 ld 0/c4


     7-  0 cm  /dis/3.   a @ b  =  a,        2 @ 4  =  2,        7 @ 3  =  7.
     7-  1 cc /     a o b  =  a + b,     6 o 3  =  9,        2 o 9  =  11./y/n
&    8-  0 qu 3.   a @ b  =  a,        2 @ 4  =  2,        7 @ 3  =  7.
&    8-  1


&    8-  2          a o b  =  a + b,      6 o 3  =  9,        2 o 9  =  11.
&    8-  3


&    8-  4          a @ (b o c)  =  (a @ b) o (a @ c) ?


&    8-  5 ca y
&    8-  6 ld c0/c3
&    8-  7 ad 1/c1
&    8-  8 ty      Correct.
```

Sample Programming (continued)

```
&   8-  9 wa n
&   8- 10 ld c0/c4
&   8- 11 ad 1/c2
&   8- 12 ty        Incorrect.


&   8- 13 br pr
&   8- 14 un        Please type either 'y' or 'n.'


&   9-  0 rd

&   9-  1 ep
&   9-  2 ld c0/c5
&   9-  3 ad c3/c6
&   9-  4 ad c4/c7
&   9-  5 ld 0/c3
&   9-  6 ld 0/c4
   10-  0 cm /dls/4.   a @ b  =  2*b,        4 @ 7  =  14,        5 @ 3  =  6.
```

INSTANCES BY TREATMENTS

Treatment A+B+

				Concept A	Concept B
1.	a @ b = 3*a*b,	2 @ 5 = 30,	4 @ 1 = 12.		
	a o b = a + b,	4 o 7 = 11,	6 o 2 = 8.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
2.	a @ b = 4*b,	6 @ 3 = 12,	2 @ 5 = 20.		
	a o b = a + b,	7 o 5 = 12,	4 o 5 = 9.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
3.	a @ b = 2*b,	7 @ 3 = 6,	4 @ 8 = 16.		
	a o b = a + b;	1 o 7 = 8,	8 o 5 = 13.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
4.	a @ b = sqr(b),	5 @ 9 = 3,	7 @ 4 = 2.		
	a o b = a*b,	4 o 6 = 24,	7 o 5 = 35.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
5.	f(x) = 6*x,	f(4) = 24,	f(7) = 42.		
	a o b = a + b,	3 o 9 = 12,	6 o 2 = 8.		
	f(a o b) = f(a) o f(b) ?				+
6.	a @ b = 4*a*b,	1 @ 5 = 20,	6 @ 2 = 48.		
	a o b = a + b,	6 o 3 = 9,	4 o 9 = 13.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
7.	a @ b = b,	5 @ 9 = 9,	8 @ 4 = 4.		
	a o b = a*b,	6 o 3 = 18,	4 o 8 = 32.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
8.	a @ b = 1,	7 @ 4 = 1,	8 @ 9 = 1.		
	a o b = a*b,	3 o 6 = 18,	8 o 4 = 32.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
9.	f(x) = x,	f(8) = 8,	f(5) = 5.		
	a o b = a*b,	4 o 7 = 28,	9 o 3 = 27.		
	f(a o b) = f(a) o f(b) ?				+
10.	a @ b = 0,	1 @ 4 = 0,	7 @ 3 = 0.		
	a o b = a*b,	5 o 8 = 40,	7 o 3 = 21.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	

Treatment A+B+ (continued)

				Concept A	Concept B
11.	f(x) = sqr(x),	f(4) = 2,	f(25) = 5.		
	a o b = a*b,	6 o 7 = 42,	9 o 3 = 27.		
	f(a o b) = f(a) o f(b) ?				+
12.	f(x) = 3*x,	f(4) = 12,	f(9) = 27.		
	a o b = a + b,	2 o 3 = 5,	5 o 4 = 9.		
	f(a o b) = f(a) o f(b) ?				+
13.	f(x) = 0,	f(6) = 0,	f(9) = 0.		
	a o b = a*b,	7 o 3 = 21,	6 o 9 = 54.		
	f(a o b) = f(a) o f(b) ?				+
14.	f(x) = 5*x,	f(6) = 30,	f(3) = 15.		
	a o b = a + b,	8 o 9 = 17,	9 o 5 = 14.		
	f(a o b) = f(a) o f(b) ?				+
15.	f(x) = x^2,	f(4) = 16,	f(7) = 49.		
	a o b = a*b,	6 o 3 = 18,	4 o 9 = 36.		
	f(a o b) = f(a) o f(b) ?				+
16.	f(x) = x,	f(7) = 7,	f(2) = 2.		
	a o b = a + b,	7 o 2 = 9,	5 o 9 = 14.		
	f(a o b) = f(a) o f(b) ?				+
17.	a @ b = b^2,	3 @ 5 = 25,	6 @ 2 = 4.		
	a o b = a*b,	3 o 6 = 18,	7 o 2 = 14.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
18.	a @ b = 0,	4 @ 8 = 0,	7 @ 2 = 0.		
	a o b = a + b,	1 o 5 = 6,	4 o 3 = 7.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
19.	f(x) = 1,	f(5) = 1,	f(7) = 1.		
	a o b = a*b,	3 o 6 = 18,	5 o 2 = 10.		
	f(a o b) = f(a) o f(b) ?				+
20.	f(x) = 0,	f(8) = 0,	f(1) = 0.		
	a o b = a + b,	8 o 1 = 9,	2 o 5 = 7.		
	f(a o b) = f(a) o f(b) ?				+

Treatment A±B+

				Concept A	Concept B
1.	f(x) = 0,	f(2) = 0,	f(7) = 0.		
	a o b = a + b,	3 o 8 = 11,	6 o 1 = 7.		
	f(a o b) = f(a) o f(b) ?				+
2.	f(x) = 0,	f(8) = 0,	f(3) = 0.		
	a o b = a*b,	5 o 2 = 10,	7 o 9 = 16.		
	f(a o b) = f(a) o f(b) = ?				+
3.	f(x) = x^2,	f(5) = 25,	f(3) = 9.		
	a o b = a*b,	6 o 3 = 18,	4 o 5 = 20.		
	f(a o b) = f(a) o f(b) ?				+
4.	f(x) = x,	f(9) = 9,	f(5) = 5.		
	a o b = a + b,	7 o 3 = 10,	1 0 8 = 9.		
	f(a o b) = f(a) o f(b) ?				+
5.	f(x) = sqr(x),	f(16) = 4,	f(4) = 2.		
	a o b = a*b,	3 o 8 = 24,	5 o 1 = 5.		
	f(a o b) = f(a) o f(b) ?				+
6.	a @ b = b^2,	3 @ 4 = 16,	1 @ 5 = 25.		
	a o b = a*b,	6 o 2 = 12,	4 o 7 = 28.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
7.	a @ b = 0,	6 @ 4 = 0,	5 @ 9 = 0,		
	a o b = a*b,	4 o 3 = 12,	7 o 9 = 63.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
8.	a @ b = 3*a*b,	1 @ 7 = 21,	3 @ 5 = 45.		
	a o b = a*b,	8 o 2 = 16,	6 o 6 = 54.		
	a @ (b o c) = (a @ b) o (a @ c) ?			-	
9.	f(x) = 1,	f(4) = 1,	f(9) = 1.		
	a o b = a*b,	7 o 2 = 14,	5 o 8 = 40.		
	f(a o b) = f(a) o f(b) ?				+
10.	a @ b = a,	3 @ 8 = 3,	9 @ 5 = 9.		
	a o b = a + b,	4 o 7 = 11,	5 o 1 = 6.		
	a @ (b o c) = (a @ b) o (a @ c) ?			-	

Treatment A+B+ (continued)

				Concept A	Concept B
11.	a @ b = 4*b,	5 @ 7 = 28,	8 @ 1 = 4.		
	a o b = a + b,	8 o 9 = 17,	7 o 3 = 10.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
12.	f(x) = 6*x,	f(5) = 30,	f(7) = 42.		
	a o b = a + b,	1 o 8 = 9,	5 o 2 = 7.		
	f(a o b) = f(a) o f(b) ?				+
13.	f(x) = 5*x,	f(6) = 30,	f(2) = 10.		
	a o b = a + b,	3 o 8 = 11,	7 o 5 = 12.		
	f(a o b) = f(a) o f(b) ?				+
14.	a @ b = sqr(b),	4 @ 9 = 3,	3 @ 4 = 2.		
	a o b = a*b,	3 o 8 = 24,	6 o 2 = 12.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
15.	f(x) = x,	f(6) = 6,	f(2) = 2.		
	a o b = a*b,	9 o 2 = 18,	4 o 7 = 28.		
	f(a o b) = f(a) o f(b) ?				+
16.	a @ b = 1,	3 @ 8 = 1,	9 @ 6 = 1.		
	a o b = a + b,	4 0 7 = 11,	5 o 2 = 7.		
	a @ (b o c) = (a @ b) o (a @ c) ?			-	
17.	a @ b = 4*a*b,	1 @ 5 = 20,	3 @ 4 = 48.		
	a o b = a*b,	2 o 8 = 16,	7 o 3 = 21.		
	a @ (b o c) = (a @ b) o (a @ c) ?			-	
18.	a @ b = 2*b,	3 @ 7 = 14,	6 @ 2 = 4.		
	a o b = a*b,	3 o 9 = 27,	5 o 2 = 10.		
	a @ (b o c) = (a @ b) o (a @ c) ?			-	
19.	a @ b = 0,	4 @ 9 = 0,	7 @ 2 = 0.		
	a o b = a + b,	2 o 8 = 10,	6 o 3 = 9.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
20.	f(x) = 3*x,	f(5) = 15,	f(2) = 6.		
	a o b = a + b,	4 o 8 = 12,	7 o 1 = 8.		
	f(a o b) = f(a) o f(b) ?				+

Treatment A+B±

				Concept A	Concept B
1.	a @ b = 1,	3 @ 9 = 1,	5 @ 2 = 1.		
	a o b = a*b,	4 o 7 = 28,	3 o 2 = 6.		
	a @ (b o c) (a @ b) o (a @ c) ?			+	
2.	a @ b = 3*a*b,	2 @ 8 = 48,	6 @ 2 = 36.		
	a o b = a + b,	9 o 4 = 13,	6 o 4 = 10.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
3.	a @ b = 4*a*b,	3 @ 6 = 72,	8 @ 2 = 64.		
	a o b = a + b,	9 o 4 = 13,	2 o 7 = 9.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
4.	f(x) = 3*x,	f(4) = 12,	f(7) = 21.		
	a o b = a*b,	3 o 5 = 15,	6 o 2 = 12.		
	f(a o b)=f(a) o f(b) ?				-
5.	f(x) = 6*x,	f(8) = 48,	f(2) = 12.		
	a o b = a*b,	1 o 8 = 8,	7 o 4 = 28.		
	f(a o b) = f(a) o f(b) ?				-
6.	f(x) = sqr(x),	f(4) = 2,	f(9) = 3.		
	a o b = a + b,	5 o 3 = 8,	8 o 4 = 12.		
	f(a o b) = f(a) o f(b) ?				-
7.	a @ b = 0,	3 @ 8 = 0,	6 @ 2 = 0.		
	a o b = a*b,	7 o 1 = 7,	4 o 8 = 32.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
8.	f(x) = 5*x,	f(7) = 35,	f(4) = 20.		
	a o b = a + b,	4 o 9 = 13,	8 o 3 = 11.		
	f(a o b) = f(a) o f(b) ?				+
9.	a @ b = 2*b,	3 @ 4 = 8,	7 @ 5 = 10.		
	a o b = a + b,	5 o 9 = 14,	6 o 3 = 9.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+ -	
10.	$f(x) = x^2$,	f(3) = 9,	f(5) = 25.		
	a o b = a*b,	3 o 7 = 21,	6 o 2 = 12.		
	f(a o b) = f(a) o f(b) ?				+

				Concept	
				A	B
11.	a @ b = b^2,	4 @ 6 = 36,	5 @ 3 = 9.		
	a o b = a*b,	3 o 6 = 18,	7 o 4 = 28.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
12.	a @ b = a,	3 @ 7 = 3,	5 @ 2 = 5.		
	a o b = a*b,	4 o 2 = 8,	5 o 7 = 35.		
	a @ (b o c) = (a @ b) o (a @ c) ?			-	
13.	a @ b = 4*b,	2 @ 5 = 20,	6 @ 3 = 12.		
	a o b = a + b,	6 o 3 = 9,	2 o 9 = 11.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
14.	f(x) = 0,	f(5) = 0,	f(8) = 0.		
	a o b = a + b,	2 o 3 = 5,	6 o 4 = 10.		
	f(a o b) = f(a) o f(b) ?				+
15.	f(x) = x^2,	f(8) = 64,	f(4) = 16.		
	a o b = a + b,	4 o 3 = 7,	5 o 7 = 12.		
	f(a o b) = f(a) o f(b) ?				-
16.	a @ b = 0,	3 @ 8 = 0,	5 @ 2 = 0.		
	a o b = a + b,	4 o 3 = 7,	5 o 9 = 14.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
17.	f(x) = 1,	f(4) = 1,	f(6) = 1.		
	a o b = a + b,	3 o 8 = 11,	5 o 2 = 7.		
	f(a o b) = f(a) o f(b) ?				-
18.	a @ b = sqr(b),	4 @ 9 = 3,	8 @ 4 = 2.		
	a o b = a*b,	2 o 8 = 16,	5 o 3 = 15.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
19.	f(x) = x,	f(7) = 7,	f(3) = 3.		
	a o b = a*b,	2 o 7 = 14,	5 o 2 = 10.		
	f(a o b) = f(a) o f(b) ?				+
20.	f(x) = 0,	f(2) = 0,	f(9) = 0.		
	a o b = a*b,	2 o 9 = 18,	7 o 4 = 28.		
	f(a o b) = f(a) o f(b) ?				+

Treatment A±B±

				Concept A	Concept B
1.	$f(x) = x$,	$f(4) = 4$,	$f(3) = 3$.		
	$a \circ b = a*b$,	$2 \circ 8 = 16$,	$5 \circ 3 = 15$.		
	$f(a \circ b) = f(a) \circ f(b)$?				+
2.	$f(x) = 0$,	$f(5) = 0$,	$f(1) = 0$.		
	$a \circ b = a*b$,	$3 \circ 8 = 24$,	$4 \circ 1 = 4$.		
	$f(a \circ b) = f(a) \circ f(b)$?				+
3.	$a @ b = a$,	$2 @ 4 = 2$,	$7 @ 3 = 7$.		
	$a \circ b = a + b$,	$6 \circ 3 = 9$,	$2 \circ 9 = 11$.		
	$a @ (b \circ c) = (a @ b) \circ (a @ c)$?			-	
4.	$a @ b = 2*b$,	$4 @ 7 = 14$,	$5 @ 3 = 6$.		
	$a \circ b = a*b$,	$5 \circ 2 = 10$,	$6 \circ 9 = 54$.		
	$a @ (b \circ c) = (a @ b) \circ (a @ c)$?			-	
5.	$a @ b = 4*a*b$,	$3 @ 7 = 84$,	$6 @ 2 = 48$.		
	$a \circ b = a*b$,	$1 \circ 8 = 8$,	$4 \circ 3 = 12$.		
	$a @ (b \circ c) = (a @ b) \circ (a @ c)$?			-	
6.	$a @ b = 0$,	$1 @ 9 = 0$,	$6 @ 3 = 0$,		
	$a \circ b = a + b$,	$5 \circ 3 = 8$,	$4 \circ 9 = 13$.		
	$a @ (b \circ c) = (a @ b) \circ (a @ c)$?			+	
7.	$a @ b = 4*b$,	$3 @ 8 = 32$,	$6 @ 2 = 8$.		
	$a \circ b = a + b$,	$3 \circ 4 = 7$,	$5 \circ 1 = 6$.		
	$a @ (b \circ c) = (a @ b) \circ (a @ c)$?			+	
8.	$f(x) = 1$,	$f(7) = 1$,	$f(9) = 1$.		
	$a \circ b = a + b$,	$8 \circ 3 = 11$,	$2 \circ 7 = 9$.		
	$f(a \circ b) = f(a) \circ f(b)$?				-
9.	$a @ b = 3*a*b$,	$6 @ 2 = 36$,	$3 @ 1 = 9$.		
	$a \circ b = a*b$,	$4 \circ 6 = 24$,	$7 \circ 3 = 21$.		
	$a @ (b \circ c) = (a @ b) \circ (a @ c)$?			-	
10.	$f(x) = x^2$,	$f(4) = 16$,	$f(3) = 9$.		
	$a \circ b = a + b$,	$5 \circ 2 = 7$,	$7 \circ 9 = 16$.		
	$f(a \circ b) = f(a) \circ f(b)$?				-

Treatment A±B± (continued)

				Concept A	Concept B
11.	a @ b = sqr(b),	4 @ 9 = 3,	7 @ 4 = 2.		
	a o b = a*b,	2 o 9 = 18,	6 o 2 = 12.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
12.	f(x) = 0,	f(7) = 0,	f(4) = 0.		
	a o b = a + b,	3 o 8 = 11,	5 o 3 = 8.		
	f(a o b) = f(a) o f(b) ?				+
13.	a @ b = 1,	4 @ 2 = 1,	5 @ 9 = 1.		
	a o b = a + b,	3 o 8 = 11,	5 o 4 = 9.		
	a @ (b o c) = (a @ b) o (a @ c) ?			-	
14.	a @ b = 0,	3 @ 7 = 0,	5 @ 2 = 0.		
	a o b = a*b,	2 o 9 = 18,	7 o 4 = 28.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
15.	f(x) = 6*x,	f(2) = 12,	f(7) = 42.		
	a o b = a*b,	4 o 2 = 8,	8 o 9 = 72.		
	f(a o b) = f(a) o f(b) ?				-
16.	f(x) = 5*x,	f(3) = 15,	f(9) = 45.		
	a o b = a + b,	3 o 1 = 4,	7 o 8 = 15.		
	f(a o b) = f(a) o f(b) ?				+
17.	a @ b = b^2,	9 @ 4 = 16,	2 @ 3 = 9.		
	a o b = a + b,	2 o 4 = 6,	8 o 5 = 13.		
	a @ (b o c) = (a @ b) o (a @ c) ?			-	
18.	f(x) = sqr(x),	f(9) = 3,	f(16) = 4.		
	a o b = a*b,	8 o 3 = 24,	5 o 2 = 10.		
	f(a o b) = f(a) o f(b) ?				+
19.	f(x) = sqr(x),	f(25) = 5,	f(4) = 2.		
	a o b = a + b,	6 o 9 = 15,	7 o 4 = 11.		
	f(a o b) = f(a) o f(b) ?				-
20.	f(x) = 3*x,	f(7) = 21,	f(4) = 12.		
	a o b = a*b,	4 o 8 = 32,	6 o 3 = 18.		
	f(a o b) = f(a) o f(b) ?				-

A.5 POSTTESTS (POA AND POB)

INTRODUCTION

This begins session 3, Frank.

As you should know, this is your last session on the computer terminal. The material which will be presented will be very similar to what you have already seen. Either @ and o or f(x) and o will be defined for you (each definition being followed by 2 examples). You will then be asked to answer a question. Simply follow these instructions:

1. Be very careful and take as much time as you need.
2. Each question requires a simple 'yes' or 'no' answer. You need only type 'y' for 'yes' and 'n' for 'no.'
3. After you type your answer be sure to hit the RETURN key.

You will notice that you will not be told whether your answer is CORRECT or INCORRECT. So, there will be no need for you to hit the RETURN key to go on to the next item. Hit the RETURN key and we will begin.

POSTTEST

Sample Student Interaction

1. f(x) = x, f(6) = 6, f(3) = 3.

   a o b = a + b, 2 o 7 = 9, 6 o 1 = 7.

   f(a o b) = f(a) o f(b) ?

2. a @ b = 2*a*b, 3 @ 7 = 42, 4 @ 2 = 16.

   a o b = a*b, 5 o 3 = 15, 7 o 9 = 63.

   a @ (b o c) = (a @ b) o (a @ c) ?

y

POSTTEST (continued)

Sample Programming

```
    J-  0 cm /poh/1.   f(x)  =  x,           f(6)  =  6,          f(3)  =  3.
    1-  1 cc /      a o b  =  a + b,          2 o 7  =  9,          6 o 1  =  7.
    1-  2 cc /y/n
&   2-  0 qu 1.   f(x)  =  x,            f(6)  =  6,          f(3) 2=  3.

&   2-  1         a o b  =  a + b,          2 o 7  =  9.          6 o 1  =  7.

&   2-  2         f(a o b)  =  f(a) o f(b) ?

&   2-  3 ca y
&   2-  4 ld c0/c3
&   2-  5 ad 1/c1
&   2-  6 ty

&   2-  7 wa n
&   2-  8 ld  c0/c4
&   2-  9 ad 1/c2
&   2- 1 ty

&   2- 1 br pr
&   2- 12 un       Please type either 'y' or 'n.'

    3-  0 cm /pod/2.   a @ b  =  2*a*b,          3 @ 7  =  42,          4 @ 2  =  16.
    3-  1 cc /      a o b  =  a*b,          5 o 3  =  15,          7 o 9  =  63./n/y
&   4-  0 qu 2.   a @ b  =  2*a*b,          3 @ 7  =  42,          4 @ 2  =  16.

&   4-  1         a o b  =  a*b,          5 o 3  =  15,          7 o 9  =  63.

&   4-  2         a @ (b o c)  =  (a @ b) o (a @ c) ?

&   4-  3 ca n
&   4-  4 ld c0/c3
&   4-  5 ad 1/c1
&   4-  6 ty
```

POSTTEST (continued)

Sample Programming (continued)

```
&    4-  7 wa y
&    4-  8 ld c0/c4
&    4-  9 ad 1/c2
&    4- 10 ty



&    4- 11 br pr
&    4- 12 un       Please type either 'y' or 'n.'
```

Posttest (POA and POB)

				Concept A	Concept B
1.	f(x) = x,	f(6) = 6,	f(3) = 3.		
	a o b = a + b,	2 o 7 = 9,	6 o 1 = 7.		
	f(a o b) = f(a) o f(b) ?				+
2.	a @ b = 2*a*b,	3 @ 7 = 42,	4 @ 2 = 16.		
	a o b = a*b,	5 o 3 = 15,	7 o 9 = 63.		
	a @ (b o c) = (a @ b) o (a @ c) ?			-	
3.	a @ b = 0,	4 @ 8 = 0,	7 @ 4 = 0.		
	a o b = a*b,	5 o 3 = 15,	2 o 8 = 16.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
4.	f(x) = 0,	f(9) = 0,	f(2) = 0.		
	a o b = a + b,	3 o 8 = 11,	4 o 2 = 6.		
	f(a o b) = f(a) o f(b) ?				+
5.	f(x) = 3,	f(9) = 3,	f(1) = 3.		
	a o b = a*b,	2 o 7 = 14,	6 o 3 = 18.		
	f(a o b) = f(a) o f(b) ?				-
6.	a @ b = b,	4 @ 9 = 9,	8 @ 2 = 2.		
	a o b = a + b,	5 o 4 = 9,	3 o 8 = 11.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
7.	a @ b = sqr(a),	4 @ 8 = 2,	9 @ 5 = 3.		
	a o b = a + b,	1 o 7 = 8,	8 o 4 = 12.		
	a @ (b o c) = (a @ b) o (a @ c) ?			-	
8.	f(x) = 1,	f(7) = 1,	f(4) = 1.		
	a o b = a*b,	3 o 6 = 18,	5 o 2 = 10.		
	f(a o b) = f(a) o f(b) ?				+
9.	f(x) = x^3,	f(2) = 8,	f(3) = 27.		
	a o b = a*b,	3 o 4 = 12,	5 o 2 = 10.		
	f(a o b) = f(a) o f(b) ?				+

Posttests (continued)

				Concept A	Concept B
10.	a @ b = a^2,	3 @ 5 = 9,	2 @ 7 = 4.		
	a o b = a + b,	6 o 2 = 8,	8 o 9 = 17.		
	a @ (b o c) = (a @ b) o (a @ c) ?			-	
11.	a @ b = 5*a*b,	3 @ 2 = 30,	4 @ 1 = 20.		
	a o b = a + b,	6 o 3 = 9,	2 o 8 = 10.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
12.	a @ b = 3*a,	5 @ 2 = 15,	7 @ 9 = 21.		
	a o b = a*b,	5 o 4 = 20,	6 o 7 = 42.		
	a @ (b o c) = (a @ b) o (a @ c) ?			-	
13.	f(x) = 7*x,	f(6) = 42,	f(4) = 28.		
	a o b = a + b,	3 o 5 = 8,	6 o 7 = 13.		
	f(a o b) = f(a) o f(b) ?				+
14.	f(x) = 4*x,	f(8) = 32,	f(2) = 8.		
	a o b = a*b,	4 o 7 = 28,	5 o 2 = 10.		
	f(a o b) = f(a) o f(b) ?				-
15.	f(x) = x - 1,	f(9) = 8,	f(4) = 3.		
	a o b = a + b,	2 o 7 = 9,	6 o 1 = 7.		
	f(a o b) = f(a) o f(b) ?				-
16.	a @ b = [illegible],	4 @ 3 = 1,	6 @ 9 = 1.		
	a o b = a*b,	3 o 7 = 21,	5 o 3 = 15.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
17.	a @ b = 3*b,	2 @ 8 = 24,	6 @ 3 = 9.		
	a o b = a + b,	6 o 3 = 9,	5 o 7 = 12.		
	a @ (b o c) = (a @ b) o (a @ c) ?			+	
18.	f(x) = 3*x^2,	f(2) = 12,	f(3) = 27.		
	a o b = a + b,	5 o 3 = 8,	6 o 7 = 13.		
	f(a o b) = f(a) o f(b) ?				-

Concept

A B

Posttests (continued)

19. a @ b = 3,
    a o b = a*b,
    a @ (b o c) = (a @ b) o (a @ c) ?

4 @ 2 = 3,
    4 o 3 = 12,

7 @ 9 = 3.
    8 o 9 = 72.

20. f(x) = sqr(3*x),
    a o b = a*b,
    f(a o b) = f(a) o f(b) ?

f(3) = 3,
    3 o 8 = 24,

f(12) = 6.
    6 o 2 = 12.

A.6. GLOSSARY OF VARIABLE NAMES

Achievement Variables

Name	Description
PCA	Pretest, Calculations with binary operations
PCB	Pretest, Calculations with functions
TA	Treatment, Concept A
TB	Treatment, Concept B
POA	Posttest, Concept A
POB	Posttest, Concept B

Temporal Variables

Name	Description
PCSIA	Stimulus Intervals for PCA
PCSIB	Stimulus Intervals for PCB
TSIA	Stimulus Intervals for TA
TSIB	Stimulus Intervals for TB
TPIA	Postfeedback Intervals for TA
TPIB	Postfeedback Intervals for TB
POSIA	Stimulus Intervals for POA
POSIB	Stimulus Intervals for POB

A.7 CORRELATION MATRIX FOR ALL VA ABLES

TABLE XVII

CORRELATION MATRIX FOR ALL VARIABLES

Variable	PCA	PCB	POB	POA	TB	TA	PCSIA	PCSIB	POSIB	POSIA	TSIB	TSIA	TPIB	TPIB
PCA	1.00	.43	.20	.25	.18	.37	-.20	.09	.15	.17	.15	.20	.08	-.15
PCB		1.00	.28	.34	.14	.32	-.24	-.12	.24	.31	.17	.22	.06	-.08
POB			1.00	.69*	.15	.31	-.01	-.02	.42	.48	.45	.48	.30	.15
POA				1.00	.17	.38	-.05	-.06	.38	.53	.37	.44	.18	.09
TB					1.00	.42	.03	.09	.09	.16	.17	.05	-.10	-.08
TA						1.00	-.16	.11	.27	.24	.15	.17	-.01	-.06
PCSIA							1.00	.69*	.24	.20	.38	.34	.01	.06
PCSIB								1.00	.29	.20	.28	.35	.01	.02
POSIB									1.00	.81*	.50*	.70*	.13	.16
POSIA										1.00	.53*	.64*	.17	.11
TSIB											1.00	.75*	.27	.22
TSIA												1.00	.30	.24
TPIB													1.00	.35
TPIA														1.00

*$p < .05$, using Sheffé procedure for multiple comparisons ($J = 14$) (i.e., $|Z| / \sqrt{1/(N-3)} \geq \sqrt{(J-1)1.96}$)
Note: See Appendix A.6 for Glossary of Variable Names.

A.8 PUBLICATIONS BASED ON FINDINGS

Shumway, Richard J. Negative instances and the acquisition of the mathematical concepts of distributivity and homomorphism. A paper submitted for presentation at the annual meeting of the American Educational Research Association, New Orleans, February, 1972.

Shumway, Richard J. Journal article to be submitted to *Journal of Educational Psychology* or *Journal of Experimental Psychology*.

Shumway, Richard J. Informal article suggesting applications of research for the classroom. Invited for submission to *The Arithmetic Teacher*.